# A THEORY FOR THE SCALAR ROUGHNESS AND THE SCALAR TRANSFER COEFFICIENTS OVER SNOW AND SEA ICE

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Abstract. Although the bulk aerodynamic transfer coefficients for sensible  $(C_H)$  and latent  $(C_E)$  heat over snow and sea ice surfaces are necessary for accurately modeling the surface energy budget, they have been measured rarely. This paper, therefore, presents a theoretical model that predicts neutral-stability values of  $C_H$  and  $C_E$  as functions of the wind speed and a surface roughness parameter. The crux of the model is establishing the interfacial sublayer profiles of the scalars, temperature and water vapor, over aerodynamically smooth and rough surfaces on the basis of a surface-renewal model in which turbulent eddies continually scour the surface, transferring scalar contaminants across the interface by molecular diffusion. Matching these interfacial sublayer profiles with the semi-logarithmic inertial sublayer profiles yields the roughness lengths for temperature and water vapor. When coupled with a model for the drag coefficient over snow and sea ice based on actual measurements, these roughness lengths lead to the transfer coefficients.  $C_E$  is always a few percent larger than  $C_H$ . Both decrease monotonically with increasing wind speed for speeds above 1 m s<sup>-1</sup>, and both increase at all wind speeds as the surface gets rougher. Both, nevertheless, are almost always between  $1.0 \times 10^{-3}$  and  $1.5 \times 10^{-3}$ .

## **Symbols**

=  $\alpha_H kG \operatorname{Pr}^{1/2} R_*^{1/4}$ , a parameter in the equations modeling flow over a rough surface, A = 10, a constant that relates eddy contact time to distance traveled, a  $b_0, b_1, b_2$  coefficients of the polynomials that predict  $z_T/z_0$  and  $z_O/z_0$  as functions of roughness Reynolds number.  $C_D$  $C_E$  $C_H$ drag coefficient at neutral stability, bulk transfer coefficient for latent heat at neutral stability, bulk transfer coefficient for sensible heat at neutral stability,  $C_p$ D specific heat of air at constant pressure, molecular diffusivity of heat, D, molecular diffusivity of water vapor, G = 5.6, a constant that relates the Kolmogorov time scale to  $t_r$ , g acceleration of gravity, h the height at which inertial and interfacial sublayer profiles match,  $H_L$ latent heat flux, H<sub>s</sub> sensible heat flux, K a function of  $\eta_s$ , defined by Equation (44), k = 0.4, von Kármán's constant K<sub>E</sub> turbulent diffusivity of water vapor at neutral stability, K<sub>H</sub> turbulent diffusivity of heat at neutral stability, K<sub>M</sub> turbulent diffusivity of momentum at neutral stability, k<sub>s</sub> intrinsic permeability of a snow cover,  $L_s$ latent heat of sublimation of ice, Pr = v/D, Prandtl number, Q water vapor density,

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Q.	water vapor density at an arbitrary reference height r,			
$\tilde{Q}_0$	water vapor density at the surface.			
<i>q</i> _	$= -H_{\star}/L_{\star}\mu$			
r	reference height.			
R.,	$= u_{1} z_{0}/v_{1}$ , roughness Revnolds number.			
s*	average profile value of an arbitrary scalar,			
So	value of the arbitrary scalar at the surface.			
~0 \$	equivalent to $t$ or $a$ , for the arbitrary scalar			
se Sc	$= \sqrt{D}$ . Schmidt number.			
T	notential temperature.			
,	time			
7	average eddy contact time over a smooth surface			
	hulk temperature just above the interfacial sublayer			
	notential temperature at an arbitrary reference level r			
1,	$- G^2(r, y y) = \frac{1}{r}$ fundamental eddy time-scale over a rough surface			
<i>i</i> ,	- $\sigma (z_0 \eta u_{*})^{-1}$ , fundamental eddy time scale over a smooth surface.			
	$-65 \eta u_{*}$ , initialitental equy lifte scale over a should sufface,			
<i>I</i> <sub>0</sub>	surface temperature,			
[*	$= -\Pi_s / \rho c_p u_{\star},$			
U	iongitudinai velocity,			
υ,	velocity at an aroltrary reference neight r,			
$U_{10}$	velocity at a reference level of 10 m, $(-1)^{1/2}$ detector reference level of 10 m,			
<sup>u</sup> *	$= (t/\rho)^{1/2}$ , including velocity,			
V	volume of air expensed by percolating ment water,			
v	downward velocity of the wetting front,			
x	downwind distance,			
$x_0$	distance over which an edgy remains in contact with a smooth surface,			
Ζ	neight,			
$^{Z}Q$	roughness length for water vapor,			
$Z_s$	roughness length for an arbitrary scalar,			
$z_T$	roughness length for temperature,			
$Z_0$	roughness length for velocity, $5.47 \times 106 \times 10^{-1}$ so substant in the expression for the velocity of the wetting front			
α	$= 5.47 \times 10^{6}$ m s <sup>-1</sup> s <sup>-1</sup> , a constant in the expression for the velocity of the wetting from,			
$\alpha_E$	$= K_E/K_M$ , inverse of the turbulent Schmidt humber,			
$\alpha_H$	= $K_H/K_M$ , inverse of the turbulent Pranot number,			
ß	the multiplicative constant in Charnock's (1955) equation, $(D_{1})^{1/2}$			
$\delta_T$	$= (DI_r)^{1/2}$ , a fundamental length scale for now over a rough surface,			
3	the dissipation rate of turbulent kinetic energy,			
Ç	$= z/\sigma_T$ , nondimensional neight over a rough surface,			
ζ	$= h/\delta_T$ , nondimensional matching neight over a rough surface,			
η	$= u_{*}^{2} z^{2}/9vDx$ , nondimensional variable characterizing now over a smooth surface,			
$\eta_s$	$= u_* z^3/9 a v D t_s$ , nondimensional neight over a smooth surface,			
$\eta_s$	$= u_{*}h^{r}/9avDt_{s}$ , nondimensional matching height over a smooth surface,			
$\eta_0$	$= u_{*}^{2} z^{3}/9 v D x_{0}$ , another form of the hondimensional height over a smooth surface,			
$\theta_i$	irreducible water content of a snow cover,			
$\theta_m$	maximum water content of a snow cover,			
v	kinematic viscosity of air,			
ς	root-mean-square surface elevation in centimeties,			
ρ	density of air,			
$ ho_i$	density of ice,			
$\rho_s$	density of show,			
σ	ratio of kinematic viscosity to indicentar diffusivity, equivalent to 11 for hear and be for water			
-	vapur,			
i di	porosity of snow.			
Ψ d	distribution function for eddy contact time over a rough surface.			
Ψr Φ	distribution function for eddy contact time over a smooth surface.			
$\Psi s$				

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#### 1. Introduction

In the atmospheric surface layer at neutral stability, the velocity (U), potential temperature (T), and water vapor density (Q) profiles have the familiar, semi-logarithmic form,

$$\frac{U(z)}{u_{*}} = k^{-1} \ln(z/z_0), \qquad (1)$$

$$\frac{T(z) - T_0}{t_*} = (\alpha_H k)^{-1} \ln (z/z_T), \qquad (2)$$

$$\frac{Q(z) - Q_0}{q_*} = (\alpha_E k)^{-1} \ln(z/z_Q).$$
(3)

Here z is the height above the surface; k is von Kármán's constant (0.4);  $T_0$  is the surface temperature;  $Q_0$  is the water vapor density of air at the snow or sea ice surface; and  $\alpha_{H}$  (=  $K_{H}/K_{M}$ ) and  $\alpha_{E}$  (=  $K_{E}/K_{M}$ ) are the ratios of the scalar turbulent diffusivities,  $K_{H}$  and  $K_{E}$ , to the turbulent diffusivity for momentum,  $K_{M}$  (e.g., Dyer, 1974). The  $u_{*}$ ,  $t_{*}$ , and  $q_{*}$  relate the profiles to the turbulent surface fluxes of momentum ( $\tau$ ) and sensible ( $H_{s}$ ) and latent ( $H_{L}$ ) heat:

$$\tau = \rho u_{\star}^2 , \qquad (4)$$

$$H_s = -\rho c_p u_* t_* , \qquad (5)$$

$$H_L = -L_s u_* q_* , \qquad (6)$$

where  $\rho$  is the air density;  $c_p$ , the specific heat of air at constant pressure; and  $L_s$ , the latent heat of sublimation of ice.

Equations (1)-(3) define the roughness lengths.  $z_0$  is the familiar roughness length for wind speed;  $z_T$  and  $z_Q$  are the roughness lengths for temperature and water vapor – the so-called scalar roughness lengths.  $z_0$  is the height at which the semi-logarithmic velocity profile extrapolates to U = 0. Similarly,  $z_T$  and  $z_Q$  are the heights at which the semi-logarithmic temperature and water vapor profiles extrapolate to the surface values,  $T_0$  and  $Q_0$ , respectively. All are fictitious levels since the semi-logarithmic profiles are not valid down to the roughness lengths.

Knowing the roughness lengths is equivalent to knowing the bulk-aerodynamic transfer coefficients for momentum  $(C_D)$ , the drag coefficient) and for the scalars, sensible  $(C_H)$  and latent  $(C_E)$  heat. After specifying a reference height r (henceforth taken as 10 m) to be the level where average values of wind speed  $(U_r)$ , temperature  $(T_r)$ , and humidity  $(Q_r)$  are measured, we define these transfer coefficients as

$$\tau = \rho C_D U_r^2,\tag{7}$$

$$H_s = \rho c_p C_H U_r (T_0 - T_r), \qquad (8)$$

$$H_L = L_s C_E U_r (Q_0 - Q_r) \,. \tag{9}$$

For neutral stability Equations (1)–(6), in turn, relate these coefficients to the roughness lengths:

$$C_{D} = \frac{k^{2}}{\left[\ln\left(r/z_{0}\right)\right]^{2}} , \qquad (10)$$

$$C_{H} = \frac{\alpha_{H} k C_{D}^{1/2}}{k C_{D}^{-1/2} - \ln(z_{T}/z_{0})},$$
(11)

$$C_E = \frac{\alpha_E k C_D^{1/2}}{k C_D^{-1/2} - \ln(z_Q/z_0)} .$$
(12)

Since correcting the transfer coefficients for stability effects is straightforward (e.g., Deardorff, 1968; Large and Pond, 1982; Andreas and Murphy, 1986), from here on all references to transfer coefficients will be to these neutral-stability ones. Clearly, from (10)–(12),  $C_H = C_D$  only when  $z_T = z_0$  and  $\alpha_H = 1$ ; and  $C_E = C_D$  only when  $z_Q = z_0$  and  $\alpha_E = 1$ . We shall see shortly that, contrary to the common assumption (e.g., Paulson, 1970; Businger *et al.*, 1971; Lettau, 1979),  $z_T$  and  $z_Q$  rarely equal  $z_0$ .

A goal of micrometeorology is to find  $C_D$ ,  $C_H$ , and  $C_E$  or, equivalently,  $z_0$ ,  $z_T$ , and  $z_Q$ . These are fairly well known over the ocean but are still only poorly known over most other horizontally homogeneous surfaces. In particular, only  $C_D$  is well known over sea ice. The extensive set of  $C_D$  values that Banke *et al.* (1980) reported show that over sea ice,  $C_D$  is an increasing function of surface roughness. The roughness parameter here is the root-mean-square (r.m.s.) surface elevation along a line parallel to the wind direction and should not be confused with the roughness length  $z_0$ . Leavitt *et al.* (1977) and Shirasawa (1981) also found that  $C_D$  increased as the sea ice surface became rougher. Kondo and Yamazawa (1986) made similar observations over snow-covered ground. Arya (1973, 1975) had theoretically predicted this increase in  $C_D$  with roughness, showing it to be a consequence of the form drag.

There have been few published attempts, however, to measure  $C_H$  and  $C_E$  over snow or sea ice. Hicks and Martin (1972) found  $C_H = 0.9 \pm 0.3 \times 10^{-3}$  and  $C_E = 2.5 \pm 0.5 \times 10^{-3}$  over snow-covered Lake Mendota. Thorpe *et al.* (1973) reported  $C_H = 1.2 \pm 0.7 \times 10^{-3}$  and  $C_E = 0.55 \pm 0.23 \times 10^{-3}$  for measurements over sea ice in the Beaufort Sea. Thus, one experiment suggested that  $C_H/C_E$  was in the interval 0.2–0.6, while the other suggested it was in the interval 0.6–0.9. Very recent measurements by Kondo and Yamazawa (1986) over snow-covered fields in Japan yielded  $C_H$  values generally in the interval  $1.1-1.5 \times 10^{-3}$ , thus supporting these earlier  $C_H$  measurements. But the value of  $C_E$  is still in question. Consequently, without an adequate theory from which to estimate  $C_H$  and  $C_E$  and, until recently, having only the seemingly contradictory values of Hicks and Martin (1972) and Thorpe *et al.* (1973), sea ice modelers have had to rely on intuition or convention. Most (e.g., Parkinson and Washington, 1979; Hibler, 1980) have followed Maykut (1978) and assumed that  $C_H$ and  $C_E$  are constant – both equal to  $1.75 \times 10^{-3}$ . But (11) and (12) imply that  $C_H$  and  $C_E$  are not constant; they depend on the characteristics of the surface and on the wind speed, since  $C_D$  depends on the surface characteristics.

This paper presents a theoretical model for predicting  $C_H$  and  $C_E$  over snow and sea ice that relies on the empirical dependence of  $C_D$  on surface roughness reported by Banke *et al.* (1980). From (11) and (12) it is clear that predicting  $C_H$  and  $C_E$  requires also finding  $z_T/z_0$  and  $z_Q/z_0$ . To do this I derive the scalar profiles in the interfacial sublayer over both aerodynamically rough and aerodynamically smooth surfaces. Matching these to the semi-logarithmic or inertial sublayer (Tennekes and Lumley, 1972, p. 147) profiles, (2) and (3), then yields the scalar roughness. I treat snow and sea ice with the same model because sea ice is generally snow covered; the two surfaces are, therefore, aerodynamically similar.

#### 2. Aerodynamically Rough Surface

With the ratio of kinematic viscosity to molecular diffusivity ( $\sigma$ ), the roughness Reynolds number  $R_* = u_* z_0 / v$  customarily parameterizes wall-bounded shear flows. Although two flows may have different velocity or length scales or different kinematic viscosities (v), they are dynamically similar if their roughness Reynolds numbers and their  $\sigma$ values are the same. Three dynamic regimes are possible, each characterized by a different  $R_*$  range (e.g., Businger, 1973). If  $R_* \leq e^{-2} = 0.135$ , the surface roughness elements are imbedded in the viscous sublayer and the surface is aerodynamically smooth. If  $R_* \geq 2.5$ , the roughness elements poke through the viscous sublayer and the surface is aerodynamically rough. For  $0.135 < R_* < 2.5$ , the surface is in transition.

Brutsaert (1975a) and Liu *et al.* (1979) based models of the turbulent transfer over aerodynamically rough surfaces on a surface-renewal model (Danckwerts, 1970, p. 100). Small eddies continually sweep into the interfacial sublayer, remain in contact with the surface for a short time, transferring heat and moisture by molecular diffusion, and then finally burst upward ahead of inrushing new eddies. Grass (1971) suggested that while the eddies are in contact with the surface, they may be stagnant – trapped by the roughness elements. Brutsaert (1975a) thus assumed that over a rough surface, the interfacial transfer of scalar properties is strictly a diffusion process. That is, using temperature as an example,

$$\frac{\partial T}{\partial t} = D \, \frac{\partial^2 T}{\partial z^2} \,, \tag{13}$$

where t is time. In (13) and in all that follows, we could use water vapor or any other scalar that obeys the same conservation equation as temperature (Hill, 1978); the only changes would be in the molecular diffusivity D and in the other thermodynamic constants, such as the  $\alpha$ 's or  $c_p$  and  $L_s$ . The boundary conditions on (13) are

$$T = T_{b} \text{ for } z > 0, \quad t = 0,$$
  

$$T = T_{b} \text{ for large } z, \quad t > 0,$$
  

$$T = T_{0} \text{ for } z = 0, \quad t > 0,$$
  
(14)

where  $T_b$  is the 'bulk' temperature above the interfacial sublayer. Many standard texts show how to solve (13) with the boundary conditions (14) (e.g., Duff and Naylor, 1966, p. 118); the solution is

$$T(z, t) = (T_0 - T_b) \operatorname{erfc}\left[\frac{z}{2(Dt)^{1/2}}\right] + T_b, \qquad (15)$$

where erfc is one minus the error function, erf (Abramowitz and Stegun, 1965, p. 297).

Equation (15) models the diffusion into a single eddy. Because eddies continually sweep over the surface, we must integrate in time to find the average interfacial sublayer temperature profile. Brutsaert (1975a) and Liu and Businger (1975) used Danckwerts's (1951) distribution function,

$$\phi_r(t) = t_r^{-1} \exp(-t/t_r), \quad t > 0,$$
(16)

to model the fraction of surface area that has had eddies in contact for a time t. Here  $t_r$  is a time-scale yet to be specified. The time-averaged temperature profile is thus

$$T(z) = \int_{0}^{\infty} T(z,t)\phi_{r}(t) dt. \qquad (17)$$

Abramowitz and Stegun (1965, p. 303) show how to integrate the error function in (15); the solution of (17) is thus

$$T(z) = T_0 - (T_0 - T_b) \left[ 1 - \exp(-z/\delta_T) \right],$$
(18)

where  $\delta_T = (Dt_r)^{1/2}$ .

Khundzhua and Andreyev (1974) verified (18) experimentally in the aqueous sublayer in the Black Sea and related the length scale  $\delta_T$  to the sensible heat flux. For air this relation is

$$\delta_T = \rho c_p D (T_0 - T_b) / H_s \,. \tag{19}$$

Actually, (19) is a necessary consequence of (18).  $H_s$  is related to the temperature gradient evaluated at the surface by

$$H_s = -\rho c_p D \left. \frac{\partial T}{\partial z} \right|_{z=0}.$$
 (20)

But from (18),  $\partial T/\partial z|_{z=0}$  is simply  $(T_b - T_0)/\delta_T$ ; (19) thus follows from (20). Values of  $\delta_T$  over snow are typically in the range 0.01 to 4 cm.

Substituting (5) for  $H_s$  in (19), we can write

$$T_0 - T_b = -u_* t_* \delta_T / D . \tag{21}$$

Substituting this in (18) yields a form for the temperature profile in the interfacial

sublayer that is compatible with the inertial sublayer profile,

$$\frac{T(z) - T_0}{t_*} = (u_* \delta_T / D) \left[ 1 - \exp(-\zeta) \right].$$
(22)

Here  $\zeta = z/\delta_T$ .

Brutsaert (1975a) and Liu *et al.* (1979) assumed that the time scale  $t_r$  in  $\delta_T$  is proportional to the Kolmogorov time-scale  $(v/\varepsilon)^{1/2}$ , where  $\varepsilon$  is the dissipation rate of turbulent kinetic energy. Formalizing this assumption by defining a proportionality constant G and setting  $\varepsilon = u_{\star}^3/z_0$  (Liu *et al.*, 1979), we have

$$t_r = G^2 (z_0 v/u_*^3)^{1/2}, \qquad (23)$$

or

$$\delta_T / z_0 = G \operatorname{Pr}^{-1/2} R_*^{-3/4}, \qquad (24)$$

where Pr = v/D is the Prandtl number. Liu *et al.* (1979) estimated G = 9.3 on the basis of data reported by Mangarella *et al.* (1973) for flow over wind waves. Since snow or sea ice surfaces are less compliant than water, this value of G may not be appropriate. In fact, in evaluating G, Liu *et al.* (1979, their Figure 3) also considered data reported by Chamberlain (1968) that imply G = 5.6 for flows over various solid surfaces. My model, with this G value, fits data collected over various solid surfaces much better than with G = 9.3. Evidently, the proportionality constant in (23) and (24) depends on whether the surface is firm or compliant.

With (24) we can now simultaneously solve (2) and (22) to find  $z_T$ . This is where I diverge from Brutsaert (1975a). I will simply match the two temperature profiles and their first derivatives at an intermediate, unknown level z = h. The temperature and the heat flux will, therefore, both be continuous from the interfacial to the inertial sublayer. With these profile and first-derivative equations, we can find the two unknowns h and  $z_T$ . Brutsaert (1975a) never computed the interfacial sublayer profile; instead he solved for  $z_T$  by matching the interfacial and inertial sublayer fluxes at the arbitrary level  $h = 7.39z_0$ . Although they do not say so explicitly, Liu *et al.* (1979) used the method of solution that I am proposing. They, however, set G = 9.3 and  $\alpha_H = \alpha_E = 1.14$ , while I use G = 5.6 and  $\alpha_H = \alpha_E = 1.0$ .

Matching the profiles (2) and (22) at z = h gives

$$\ln \zeta - \ln (z_0/\delta_T) - \ln (z_T/z_0) = A[1 - \exp(-\zeta)], \qquad (25)$$

where  $\hat{\zeta} = h/\delta_T$ , and

$$A = \alpha_H k G \Pr^{1/2} R_*^{1/4} .$$
 (26)

Matching the first derivatives at  $\hat{\zeta}$  yields

$$\hat{\zeta} \exp\left(-\hat{\zeta}\right) = A^{-1}. \tag{27}$$

We need not worry here or in (25) about the stability of the atmospheric surface layer and its effects on the inertial sublayer profiles. We shall see that the matching level is well below the region where atmospheric stability affects the semi-logarithmic profiles (Bradley, 1972). Notice that (27) has a solution only for A > 2.72. Substituting it into (25) gives a formal expression or  $z_T/z_0$ ,

$$z_T / z_0 = \hat{\zeta}(\delta_T / z_0) \exp[\hat{\zeta}^{-1} - A].$$
(28)

Here A and  $\hat{\delta}_T$  are functions only of Pr and  $R_*$ . I solve for  $\hat{\zeta}$  by using Newton's method to find the zeroes in (27) as functions of Pr and  $R_*$ . Figure 1 shows  $h/z_0 = (\hat{\zeta} \delta_T/z_0)$  for both temperature and water vapor as a function of  $R_*$ . The values are much smaller than the constant that Brutsaert (1975a) used. He chose  $h = 7.39z_0$  because this is approximately the height of the roughness elements. By implying that the roughness elements protrude above the interfacial sublayer, Figure 1 is, thus, consistent with our conceptual model of molecular diffusion into stagnant eddies.



Fig. 1. The matching height over an aerodynamically rough surface as a function of roughness Reynolds number. For temperature,  $\sigma = 0.71$ ; for water vapor,  $\sigma = 0.63$ .

Figure 2 shows the transition of temperature and water vapor profiles from the interfacial to the inertial sublayer for  $R_* = 10$ . Both the profiles and their first derivatives are continuous. The vertical fluxes of sensible and latent heat are, therefore, also continuous.

Figure 3 compares predictions of my model for the scalar roughness,  $z_s$ , with experimental data collected over aerodynamically rough surfaces ( $R_* \ge 2.5$ ) by Owen and Thomson (1963) and Chamberlain (1966, 1968). All the data sets are from wind tunnel measurements and represent three different  $\sigma$  values. Chamberlain (1966) studied water vapor transfer over toweling and artificial grass ( $\sigma = 0.62$  at  $\sim 20$  °C). Because



Fig. 2. The matching of interfacial and inertial sublayer profiles of temperature ( $\sigma = 0.71$ ) and water vapor ( $\sigma = 0.63$ ) over an aerodynamically rough surface for  $R_{\star} = 10$ .

the grass, however, had a roughness length of 1.0 cm – much larger than that typical of snow or sea ice (Untersteiner and Badgley, 1965; Banke *et al.*, 1980; Schmidt, 1982) – and roughness elements unlike those of snow or sea ice, Figure 3 omits those data. Chamberlain (1968) collected his thorium-B ( $\sigma = 2.78$ ) and water vapor data over a host of two- and three-dimensional roughness elements, some with roughness lengths as large as 0.6 cm. The figure indicates the data for surfaces with  $z_0 > 0.2$  cm – roughly the maximum sea ice roughness that Banke *et al.* (1980) reported. No systematic difference between large and small roughness lengths is evident, however. Owen and Thomson (1963) looked at the transfer of camphor ( $\sigma = 3.2$ ) over glass surfaces with two- and three-dimensional roughness.

Measuring  $z_s/z_0$  is difficult under any circumstances – even in a wind tunnel – because, as (1) and (2) show with temperature for example, we have to know  $u_*$ ,  $t_*$ , and  $z_0$ . Chamberlain (1968) explained that his  $z_0$  values alone may have been in error by 50%.



Fig. 3. Model predictions for an aerodynamically rough surface compared with measured scalar roughness lengths for water vapor, thorium-B, and camphor.

The scatter of the data in Figure 3 is, thus, not surprising. In view of this uncertainty, the model predictions are quite reasonable. The model reproduces the  $R_*$  dependence at constant  $\sigma$  very well and has  $z_s/z_0$  decreasing with increasing  $\sigma$ , as the data do. Only the  $z_s/z_0$  data from Owen and Thomson (1963), for which  $\sigma = 3.2$ , deviate significantly from model predictions.

Dipprey and Sabersky (1963) investigated heat transfer in water-filled pipes. Figure 4 compares model predictions of  $z_s/z_0$  with data for which  $R_* \ge 2.5$  obtained from their Figures 6, 7, and 8. Pipe flow may not seem to be a good experimental model for flow in the atmospheric surface layer, but the two are, in fact, mathematically equivalent. Flow in pipes is characterized by a viscous sublayer near the wall and a semi-logarithmic inertial sublayer farther from the wall (Schlichting, 1968, p. 578; Tennekes and Lumley, 1972, p. 149), just like my model for the atmospheric surface layer. Figure 4 confirms the validity of the comparison. Dipprey and Sabersky (1963) varied  $\sigma$  by changing the water temperature; the model fits their data extremely well for  $\sigma = 1.20$  and reasonably



Fig. 4. Model predictions for an aerodynamically rough surface compared with the experimental data of Dipprey and Sabersky (1963).

well at the other  $\sigma$  values. For all of the  $\sigma$  values in the figure, the difference between the data and the model predictions tends to decrease as  $R_*$  increases. This suggests some experimental imprecision at low flow rates or a possible inadequacy in the model for  $2.5 \le R_* < 10$ . Fortuitously, this low- $R_*$  bias is small for  $\sigma \simeq 0.6-1.2$ , the range most relevant for heat and moisture transfer over snow. Therefore, my model seems to be an adequate fit to the available data that are most representative of an aerodynamically rough snow or sea ice surface.

### 3. Aerodynamically Smooth Surface

Although natural surfaces are seldom aerodynamically smooth, I model scalar transfer over a smooth surface for completeness and so I can predict the transfer over a transitional surface by interpolating between smooth and rough regimes.

Brutsaert (1975a) modeled the transfer over a smooth surface by again postulating a surface-renewal mechanism. Over a smooth surface, however, an impinging eddy remains in motion; an advective diffusion model, thus, governs the process,

$$U(z) \ \frac{\partial T}{\partial x} = D \ \frac{\partial^2 T}{\partial z^2} , \qquad (29)$$

where U(z) is the velocity profile in the viscous sublayer. The boundary conditions on (29) are

$$T = T_b \quad \text{for } z > 0, \qquad x = 0,$$
  

$$T = T_b \quad \text{for large } z, \qquad x > 0,$$
  

$$T = T_0 \quad \text{for } z = 0, \qquad x > 0.$$
(30)

In the viscous sublayer,  $zu_*/v < 5$ ,

$$U(z) = u_{*}^{2} z / v$$
 (31)

(e.g., Monin and Yaglom, 1971, p. 270; Brutsaert, 1975a). On substituting (31) into (29) and making the change of variables  $\eta = u_*^2 z^3/9 vDx$  suggested by Kestin and Persen (1962), we get the equation

$$\left(\eta + \frac{2}{3}\right)\frac{\partial T}{\partial \eta} + \eta \frac{\partial^2 T}{\partial \eta^2} = 0.$$
(32)

The solution satisfying the boundary conditions is (Kestin and Persen, 1962)

$$T(\eta) = T_0 - (T_0 - T_b) \frac{\gamma(\frac{1}{3}, \eta)}{\Gamma(\frac{1}{3})}, \qquad (33)$$

where  $\Gamma$  is the gamma function, and  $\gamma$  is the incomplete gamma function (Abramowitz and Stegun, 1965, p. 255, 260, respectively). Although Brutsaert (1975a) posed (29) with the boundary conditions (30), he solved only for  $\partial T/\partial z$  at z = 0.

Next impose the assumptions of the surface-renewal model – that the eddy is in contact with the surface only for  $0 \le x \le x_0$ , where  $x_0$  is typically 1 m. Averaging  $T(\eta)$  over this distance interval yields

$$T(z, x_0) = T_0 - \frac{(T_0 - T_b)}{\Gamma(\frac{1}{3})} \left[ \gamma(\frac{1}{3}, \eta_0) + \eta_0 \Gamma(-\frac{2}{3}, \eta_0) \right],$$
(34)

where

$$\Gamma(-\frac{2}{3},\eta_0) = \Gamma(-\frac{2}{3}) - \gamma(-\frac{2}{3},\eta_0)$$
(35)

is another form of the incomplete gamma function, and  $\eta_0 = u_*^2 z^3/9 v D x_0$ .

Removing the explicit dependence on  $x_0$  in (34) requires averaging over all possible values of  $x_0$ . Brutsaert (1975a) suggested setting

$$x_0 = a u_* t, \tag{36}$$

where a is a constant, and then using (16) for the distribution function of t. That is, we would again have the integral (17) with (34) substituted for (15). If Brutsaert had attempted this integration, he would have found the integral infinite, because the distribution function (16) is not the appropriate one over a smooth surface. The work of Kim *et al.* (1971) suggested that for smooth surfaces, the eddy-contact time has the distribution

$$\phi_s(t) = (t/t_s^2) \exp(-t/t_s), \quad t > 0,$$
(37)

where  $t_s$  is a new time-scale. The average contact time is thus  $\bar{t} = 2t_s$ . In addition, from Figures 23 and 24 in Kim *et al.* (1971) I derived

$$\bar{t} = 170 v/u_*^2$$
, (38)

or

$$t_s = 85 \, v/u_{\,\star}^2 \,. \tag{39}$$

Notice, since  $v/u_*$  is the appropriate scaling length in the viscous sublayer over a smooth surface (Tennekes and Lumley, 1972, p. 152),  $v/u_*^2$  is the only reasonable time-scale there.

Substituting (34) and (37) into the time-averaging integral, (17), rearranging arguments, and defining  $\eta_s = u_* z^3/9avDt_s$ , we derive an expression for the average profile in the interfacial sublayer,

$$T(z) = T_0 - \frac{(T_0 - T_b)\eta_s^2}{\Gamma(\frac{1}{3})} \int_0^\infty \left[\eta_0^{-1}\gamma(\frac{1}{3}, \eta_0) + \Gamma(-\frac{2}{3}, \eta_0)\right]\eta_0^{-2} \\ \times \exp(-\eta_s/\eta_0) \,\mathrm{d}\,\eta_0 \,.$$
(40)

The method of steepest descent (e.g., Dennery and Krzywicki, 1967) is useful for approximating such difficult integrals; it yields

$$T(z) = T_0 - \frac{(T_0 - T_b)}{\Gamma(\frac{1}{3})} \left\{ 0.960 \,\gamma(\frac{1}{3}, \eta_s/2) + 1.383 \,\eta_s[\gamma(\frac{1}{3}, \eta_s) - \Gamma(\frac{1}{3}) + \eta_s^{-2/3} \exp(-\eta_s)] \right\}.$$
(41)

This, to my knowledge, is the first derivation of the interfacial sublayer profile of a scalar over an aerodynamically smooth surface that is based on surface-renewal concepts.

As with the rough-surface case, we must eliminate  $T_b$  in (41) to match the interfacial and inertial sublayer profiles. Again, we know how the surface flux is related to the profile – by Equation (20). Using this and substituting (39) for  $t_s$  in (41) we find

$$T_0 - T_b = 1.519(85a)^{1/3} \operatorname{Pr}^{2/3} t_{\star}$$
 (42)

Hence,

$$T(z) = T_0 + 1.458(85a)^{1/3} \operatorname{Pr}^{2/3} t_* K(\eta_s), \qquad (43)$$

where

$$K(\eta_s) = \Gamma(\frac{1}{3})^{-1} \{ \gamma(\frac{1}{3}, \eta_s/2) + 1.44[\gamma(\frac{1}{3}, \eta_s) - \Gamma(\frac{1}{3}) + \eta_s^{-2/3} \exp(-\eta_s)] \}.$$
(44)

Notice, substituting  $t_s$  and recognizing that over a smooth surface  $z_0 = e^{-2} v/u_*$  (Tennekes and Lumley, 1972, p. 157), we can rewrite  $\eta_s$  as

$$\eta_s = \frac{e^{-6}}{9 \times 85a} \operatorname{Pr}\left(\frac{z}{z_0}\right)^3.$$
(45)

Matching the profiles at z = h or at

$$\hat{\eta}_s = \frac{e^{-6}}{9 \times 85a} \operatorname{Pr}\left(\frac{h}{z_0}\right)^3, \tag{46}$$

we get

$$\frac{1}{3}\ln\hat{\eta}_s + \frac{1}{3}\ln\left(\frac{9\times85a}{e^{-6}\,\mathrm{Pr}}\right) - \ln(z_T/z_0) = 1.458\,\alpha_H k(85a)^{1/3}\,\mathrm{Pr}^{2/3}\,K(\hat{\eta}_s)\,.$$
 (47)

And matching the slopes there, too,

$$\frac{1}{3} = 1.458 \alpha_H k (85a)^{1/3} \operatorname{Pr}^{2/3} \Gamma(\frac{1}{3})^{-1} \{ (\hat{\eta}_s/2)^{1/3} \exp(-\hat{\eta}_s/2) + 1.44 \eta_s [\gamma(\frac{1}{3}, \hat{\eta}_s) - \Gamma(\frac{1}{3}) + \frac{1}{3} \hat{\eta}_s^{-2/3} \exp(-\hat{\eta}_s) ] \}.$$
(48)

Again I solve (48) for  $\hat{\eta}_s$  by Newton's method and then find  $z_T/z_0$  from (47),

$$z_T/z_0 = \hat{\eta}_s^{1/3} \left(\frac{9 \times 85a}{e^{-6} \operatorname{Pr}}\right)^{1/3} \exp\left[-1.458 \alpha_H k (85a)^{1/3} \operatorname{Pr}^{2/3} K(\hat{\eta}_s)\right].$$
(49)

The constant *a* is yet to be specified. The viscous sublayer velocity profile, (31), is approximately valid from the surface to the lower boundary of the inertial sublayer at  $30 v/u_*$  (Tennekes and Lumley, 1972, p. 160). Therefore, integrating this profile from zero to  $30 v/u_*$  should yield an average velocity  $\overline{U}$  for the viscous sublayer. That average is  $\overline{U} = 15u_*$ . Comparing this result with (36), we see that *a* should be roughly 15. I have found that the value a = 10 fits the available data best.

Figure 5 shows model calculations of the nondimensional matching height as a

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Fig. 5. The nondimensional matching height over an aerodynamically smooth surface as a function of  $\sigma$ .

function of  $\sigma$ . For temperature and water vapor, the value  $hu_*/v$  is about 32. Brutsaert (1975a) did his matching by assuming that  $hu_*/v = 30$  for all values of  $\sigma$ . Figure 6 shows the matching of interfacial and inertial sublayer profiles for temperature and water vapor over an aerodynamically smooth surface. As with the aerodynamically rough surface, both profiles and their first derivatives are continuous at h.

Finally, Figure 7 compares model predictions with the scanty data available from flows over smooth surfaces. In the figure, the data from Chamberlain (1968) are the 'smooth surface' values from his Tables 2 and 3. The data from Dipprey and Sabersky (1963) are from their smooth pipe (E-3, their Figure 5) and from their three rough pipes (their Figures 6–8) for runs with  $R_* \leq 0.135$ . The fourth-order polynomial

$$\ln (z_s/z_0) = 0.0399 - 3.92 \ln \sigma - 1.22 (\ln \sigma)^2 - 0.254 (\ln \sigma)^3 - 0.0748 (\ln \sigma)^4$$
(50)

is a good representation of my model results for  $0.35 \le \sigma \le 10.0$ . The figure also shows Brutsaert's (1975b) prediction,

$$z_s/z_0 = \exp\left[-k(13.6\sigma^{2/3} - 13.5)\right],$$
(51)



Fig. 6. The matching of interfacial and inertial sublayer profiles of temperature ( $\sigma = 0.71$ ) and water vapor ( $\sigma = 0.63$ ) over an aerodynamically smooth surface.

and von Kármán's (Goldstein, 1965, p. 657; Monin and Yaglom, 1971, p. 342),

$$z_s/z_0 = \exp\left(-5k\{(\sigma-1) + \ln\left[1 + 0.83(\sigma-1)\right]\}\right).$$
(52)

The three models are so close that, with the scatter in the experimental data and their sparseness, it is impossible to decide which is the best predictor of  $z_s/z_0$ . Brutsaert's relation is the simplest computationally, but mine probably has the best physical basis. As we discussed, Brutsaert used a form for the distribution function  $\phi_s$  that is unsupported by wind tunnel observations over smooth surfaces. And von Kármán had to assume two matching heights: one at the top of the molecular sublayer and another at the base of the logarithmic layer. In contrast, my model yields the matching height as a function of  $\sigma$  (Figure 5).

In Figure 7 all three models predict  $z_s/z_0 \simeq 1$  for  $\sigma = 1$ . This is compatible with the Reynolds analogy – that over an aerodynamically smooth surface, where the roughness



Fig. 7. Current model predictions for an aerodynamically smooth surface compared with measured scalar roughness lengths for water vapor, thorium-B, and heat and with models by Brutsaert (1975b) and von Kármán (Goldstein, 1965).

elements cannot transfer momentum through pressure forces, the transfers must be identical for momentum and for a scalar contaminant with  $\sigma = 1$ . Von Kármán (Goldstein, 1965, p. 657) explicitly assumed the validity of the Reynolds analogy and thus forced his model to predict  $z_s/z_0 = 1$  at  $\sigma = 1$ . While neither Brutsaert (1975a, b) nor I made this assumption, his model predicts  $z_s/z_0 = 0.96$  at  $\sigma = 1$ , and mine predicts  $z_s/z_0 = 1.04$ .

### 4. Scalar Transfer Coefficients

With the results of the last two sections, we can specify  $z_T/z_0$  and  $z_Q/z_0$  over snow or sea ice for all  $R_*$  (Figure 8). Since temperatures will be 0 °C or less, the Prandtl number  $(\nu/D)$  is 0.71 and the Schmidt number  $(\nu/D_w)$  is 0.63, with values for the molecular diffusivity of water vapor,  $D_w$ , taken from Pruppacher and Klett (1978, p. 413). In  $R_*$ ,  $\nu$  is evaluated at -5 °C. Because the model predicts  $z_s/z_0$  only over aerodynamically smooth and rough surfaces, to obtain  $z_s/z_0$  values in the transition region, I did a log-log interpolation between model results at  $R_* = 0.135$  and  $R_* = 2.5$ .



Fig. 8. Model predictions of  $z_T/z_0$  and  $z_O/z_0$  over snow and sea ice.

Figure 8 shows that  $z_Q$  is slightly larger than  $z_T$  at all roughness Reynolds numbers. As Figures 3, 4, and 7 imply, this is strictly an effect of the difference in molecular diffusivities. Both  $z_T$  and  $z_Q$  are usually less than  $z_0$ ;  $z_T/z_0$  and  $z_Q/z_0$  become less than one in the transition region and decrease monotonically in the aerodynamically rough region. Thus,  $z_T$  and  $z_Q$  are virtually always less than  $z_0$  in natural flows. Several recent models predicted scalar transfer over aerodynamically rough surfaces. The model by Garratt and Hicks (1973) – which is just the empirical equation that Owen and Thomson (1963) derived – predicts z/z, values generally five times larger than my

and Thomson (1963) derived – predicts  $z_s/z_0$  values generally five times larger than my model. The predictions of  $z_s/z_0$  by Liu *et al.* (1979), which admittedly are for a water surface, are almost an order of magnitude smaller than mine. Brutsaert's (1975b) model predicts  $z_s/z_0$  values smaller than mine for  $R_* < 20$  but values fairly close to mine for larger  $R_*$ .

For facilitating computer modeling, I have fitted the model results in Figure 8 with polynomials of the form

$$\ln(z_s/z_0) = b_0 + b_1 \ln R_* + b_2 (\ln R_*)^2.$$
(53)

Table I lists the coefficients for smooth, transition, and rough surfaces.

	for temperature ( $\sigma = 0.71$ ) and water vapor ( $\sigma = 0.63$ )			
	$R_* \leq 0.135$	$0.135 < R_{*} < 2.5$	$2.5 \le R_* \le 1000$	
Tempe	rature			
$b_0$	1.250	0.149	0.317	
$b_1$	_	- 0.550	- 0.565	
$b_2$	-	-	- 0.183	
Water	vapor			
$b_0$	1.610	0.351	0.396	
$b_1$	-	- 0.628	- 0.512	
$\dot{b_2}$	_	-	-0.180	

TABLE I Values of the coefficients in the polynomials, Equation (53), that predict  $z_s/z_0$ 

As (11) and (12) show, if we know  $z_T/z_0$ ,  $z_Q/z_0$ , and  $C_D$ , we can find  $C_H$  and  $C_E$ . The model for  $C_D$  presented by Banke *et al.* (1980) synthesizes a collection of  $C_D$  values measured over various types of sea ice in the Beaufort Sea and in Robeson Channel in the Canadian Archipelago. Their empirical result,

$$10^3 C_D = 1.10 + 0.072\xi, \tag{54}$$

parameterizes the drag coefficient in terms of the r.m.s. surface roughness  $\xi$  in centimetres. Banke *et al.* (1980) found  $\xi$  by using a leveling rod to measure the surface elevation at 1-m intervals for several hundred metres upwind of their instruments. Integrating the power spectrum of these height data over wavelengths less than 13 m yielded  $\xi^2$ . Notice that through (10),  $\xi$  has a one-to-one relationship with  $z_0$ . The  $z_0$  values reported by Schmidt (1982) for blowing snow, those summarized by Chamberlain (1983) for drifting snow and sand, and those implicit in the  $C_D$  measurements over snow by Kondo and Yamazawa (1986) are generally in the range reported by Banke *et al.* (1980); hence, I assume that (54) is a valid model for snow fields, too.

Kind (1976) and Chamberlain (1983) suggested that roughness lengths for snow and sand obey Charnock's (1955) relation,

$$z_0 = \beta u_*^2 / g \,, \tag{55}$$

where g is the acceleration of gravity and  $\beta$  is a dimensionless constant with value 0.010-0.016. Through (10), (55) would yield  $C_D$ . But because (54) parameterizes the form drag, which is an important effect over sea ice, I prefer (54) to (55).

The routine for estimating  $C_H$  and  $C_E$  is first to select a  $\xi$  value; this value then defines  $C_D$  from (54).  $C_D$ , in turn, has a one-to-one relationship with  $z_0$  through (10). Finally, we compute  $R_{\star}$  from

$$R_* = U_{10} C_D^{1/2} z_0 / v, (56)$$

where  $U_{10}$  is the wind speed at 10 m. Substituting  $R_*$  into (53), we use the resulting  $z_T/z_0$  and  $z_Q/z_0$  values to compute  $C_H$  and  $C_E$  for a 10-m reference height from (11) and (12). Figures 9 and 10 show  $C_H$  and  $C_E$  as functions of  $\xi$  and  $U_{10}$ . Remember again,  $C_D$ ,  $C_H$ , and  $C_E$  are the values at neutral stability.

According to Figures 9 and 10, the predicted  $C_E$  value is always 1-3% larger than  $C_H$ . And except for very low wind speed, when the surface is aerodynamically smooth or in transition, both are smaller than  $C_D$ . Both  $C_H$  and  $C_E$  are generally between



Fig. 9. Model predictions of  $C_H$  over snow or sea ice as a function of the r.m.s. surface roughness (in centimetres) and the 10-m wind speed  $U_{10}$ . The arrows on the right show  $C_D$  for the indicated  $\xi$  value. The data points are from measurements by Kondo and Yamazawa (1986).



Fig. 10. Model predictions of  $C_E$  over snow or sea ice as a function of the r.m.s. surface roughness (in centimetres) and the 10-m wind speed  $U_{10}$ . The arrows on the right show  $C_D$  for the indicated  $\xi$  value.

 $1.0 \times 10^{-3}$  and  $1.5 \times 10^{-3}$ . Only over the roughest surfaces – and then only at low wind speeds – are  $C_H$  and  $C_E$  larger than  $1.5 \times 10^{-3}$ .  $C_H$  and  $C_E$  go below  $1.0 \times 10^{-3}$  only at unusually high wind speeds. The model predictions are therefore largely incompatible with the scalar transfer coefficients reported by Hicks and Martin (1972) and Thorpe *et al.* (1973). For wind speeds of about 3 m s<sup>-1</sup>, Hicks and Martin (1972) found average values of  $C_H$  and  $C_E$  over snow-covered Lake Mendota to be  $0.9 \times 10^{-3}$  and  $2.5 \times 10^{-3}$ , respectively. Over ice in the Beaufort Sea, Thorpe *et al.* (1973) found averages of  $C_H = 1.2 \times 10^{-3}$  and  $C_E = 0.55 \times 10^{-3}$  for winds ranging from 5 to  $10 \text{ m s}^{-1}$ . Only the  $C_H$  measurement by Thorpe *et al.* (1973) agrees with theoretical predictions.

The form of the  $C_H$  and  $C_E$  functions in Figures 9 and 10 is different from that predicted by Kondo (1975) and by Liu *et al.* (1979) for  $C_H$  and  $C_E$  over the ocean. According to Figures 9 and 10,  $C_H$  and  $C_E$  are monotonically decreasing functions of wind speed, except at very low speeds over smooth surfaces where they are constant. Kondo (1975) predicted that over the ocean  $C_H$  and  $C_E$  have minimums at about  $2 \text{ m s}^{-1}$  and then increase gradually as the wind speed increases. Liu *et al.* (1979) predicted that  $C_H$  and  $C_E$  have local minimums at roughly  $2 \text{ m s}^{-1}$ , local maximums at about  $5 \text{ m s}^{-1}$ , then decrease slowly for increasing wind speeds. The basic reason for the differences between my model and these is that over the ocean,  $C_D$  has a wind-speed dependence. No such wind-speed dependence has been established for the drag coefficient over snow or sea ice.

Figure 9 also shows  $C_H$  values derived from the measurements reported by Kondo and Yamazawa (1986). Because they reported bulk transfer coefficients referenced to 1 m, I had to obtain their raw data (J. Kondo, 1986, personal communication) to compute neutral-stability  $C_D$  and  $C_H$  values referenced to 10 m. Also, the roughness parameter that they reported reflected the microscale roughness; they determined it by measuring the snow-surface elevation at centimetre intervals for several metres. It, thus, differs from the macroscale roughness  $\xi$  that I use in the model. To associate a  $\xi$  value with each of their  $C_H$  values, I therefore converted each  $C_D$  value to its implicit  $\xi$  value using (54). This, of course, does not yield very accurate  $\xi$  values but, at least, serves to separate the  $C_H$  values according to relative roughness.

Although Kondo and Yamazawa (1986) measured too few  $C_H$  values to give the model a thorough test, the predicted  $C_H$  values in Figure 9 agree with their data. Only one of their measurements falls outside the range of  $C_H$  values predicted, and only one falls inside the range when it should be outside ( $\xi > 16$  cm). Though the  $\xi$  values assigned to the measured  $C_H$  values may not be very accurate, the measured  $C_H$  values do generally have the same trend with surface roughness as the predicted values. Consequently, the data of Kondo and Yamazawa (1986) tend to confirm that my model correctly predicts the magnitude of the neutral-stability  $C_H$  value and its dependence on wind speed and surface roughness.

Measuring  $C_H$  and  $C_E$  over natural snow and sea ice surfaces is extremely difficult. First,  $u_*$  must be measured, either by measuring the vertical velocity profile or by measuring  $\tau$  directly. Next,  $t_*$  and  $q_*$  must be measured – again, either by measuring the vertical profiles of temperature and water vapor or by measuring  $H_s$  and  $H_L$  directly. These are necessary not only for finding  $C_H$  and  $C_E$  but also for making stability corrections. Last, and probably most important, is the measurement of  $T_0$  and  $Q_0$ . Since  $T_0 - T_r$  and  $Q_0 - Q_r$  are rarely large over frozen surfaces, the  $T_0$  and  $Q_0$  measurements must be precise. But because the surface is ill-defined, simply deciding what level over the snow corresponds to  $T_0$  and  $Q_0$  is a problem; finding instruments capable of measuring  $T_0$  and  $Q_0$  without disturbing the integrity of the surface is another. Consequently, the most careful flux measurements are of little value for specifying  $C_H$  and  $C_E$  if  $T_0$  and  $Q_0$  are not measured as carefully. Andreas (1986) discussed this problem of measuring snow-surface temperature further and offered a possible solution.

### 5. Discussion

I have based my model on wind tunnel data collected over various solid surfaces. Snow, however, is a porous medium, a mixture of ice and air saturated with water vapor. If somehow that air is expelled from the snow, it could contribute to the surface sensible and latent heat fluxes in ways that I have not modeled. Water percolating down through the snow as a result of surface melting is the most likely way to expel air from a snow pack. Colbeck (1976) developed a theoretical model with which we can estimate the sensible and latent heat fluxes accompanying this percolation.

As snow melts near the surface, liquid water percolates downward as a wetting front with velocity v. The volume (V) of air expelled from the snow per unit time and per unit surface area is thus

$$\frac{\mathrm{d}V}{\mathrm{d}t} = v(\theta_m - \theta_i). \tag{57}$$

Here  $\theta_i$  is the irreducible water content of the snow, the water held in the menisci between the snow grains; and  $\theta_m$  is the maximum amount of water that freely draining snow can hold.  $\theta$  is dimensionless, being the volume of water per unit volume of snow.

 $\theta_m - \theta_i$  is largest in dry snow, since  $\theta_i$  is zero, by definition. But because the velocity of the wetting front is much higher in ripe snow than in dry snow (Colbeck, 1976), dV/dt will be largest for ripe snow. Colbeck (1976) showed that for ripe snow, the maximum velocity of the wetting front is

$$v = \alpha k_s \phi^{-3} (1 - \theta_i)^{-1} (\theta_m^2 + \theta_m \theta_i + \theta_i^2), \qquad (58)$$

where  $\alpha$  (= 5.47 × 10<sup>6</sup> m<sup>-1</sup> s<sup>-1</sup>) is a theoretical constant,  $k_s$  (= 2.0 × 10<sup>-9</sup> m<sup>2</sup>) is the intrinsic permeability of the snow (Colbeck, 1976; Colbeck and Anderson, 1982),  $\phi = (1 - \rho_s/\rho_i)$  is the porosity of the snow, and  $\rho_s$  and  $\rho_i$  are the densities of snow and ice. With typical values for  $\phi$ ,  $\theta_m$ , and  $\theta_i$  of 0.7, 0.1, and 0.03, respectively (S. C. Colbeck, 1985, personal communication), v is 4.6 × 10<sup>-4</sup> m s<sup>-1</sup> or 1.6 m hr<sup>-1</sup>.

Using this result in (57) yields  $dV/dt = 3.2 \times 10^{-5} \text{ m s}^{-1}$  (m<sup>3</sup> of air expelled per m<sup>2</sup> of surface per second). Melt water must form at the surface at a like rate; this would thus require a surface energy source of 10700 W m<sup>-2</sup>! Because this flux is at least 30 times larger than we would expect for the combined radiative and turbulent fluxes at the surface of a snow cover on a sunny day, it is unlikely that dV/dt will ever exceed  $1 \times 10^{-6} \text{ m s}^{-1}$ . Since saturated air at 0 °C contains  $4.84 \times 10^{-3} \text{ kg m}^{-3}$  water vapor, the maximum latent heat flux associated with this rate of air expulsion is 0.01 W m<sup>-2</sup>. Similarly, the sensible heat flux resulting from the expulsion of this air is  $0.001\Delta T$  W m<sup>-2</sup>, where  $\Delta T$  is the snow-air temperature difference in °C.

Since a  $\Delta T$  of 10 °C would be an uncommonly large snow-air temperature difference, both of these percolation fluxes would be inseparable from the noise in any measurement of the turbulent surface fluxes. Therefore, although snow is porous, the air and water vapor within it exchange with the free air so slowly that, for the purposes of turbulent heat exchange, snow behaves as a solid surface. Consequently, the data collected over solid surfaces that I have used to tune my model should be representative of a snow cover with similar surface roughness.

## 6. Conclusions

I have modeled the transfer of the passive scalar contaminants temperature and water vapor over aerodynamically rough and smooth snow and sea ice. The basis of the model is a smooth matching of interfacial and inertial sublayer profiles. The inertial sublayer profile has the usual semi-logarithmic form; the interfacial sublayer profiles over smooth and rough surfaces derive from a turbulent surface-renewal model. This, evidently, is the first such derivation of the interfacial sublayer profile for a passive scalar over an aerodynamically smooth surface.

The model yields values of  $z_T/z_0$  and  $z_Q/z_0$  as functions of the roughness Reynolds number. Using these values and the empirical model for the drag coefficient over sea ice given by Banke *et al.* (1980), I predict the 10-m bulk transfer coefficients for sensible  $(C_H)$  and latent  $(C_E)$  heat at neutral stability over snow and sea ice. These depend on the wind speed and on a surface roughness parameter.  $C_E$  is 1-3% larger than  $C_H$ ; at winds speeds greater than 3 m s<sup>-1</sup>, both are virtually always between  $1.0 \times 10^{-3}$  and  $1.5 \times 10^{-3}$ . Only at low wind speeds – which usually do not persist – and over very rough surfaces are  $C_H$  and  $C_E$  larger than  $1.5 \times 10^{-3}$ .

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