AVERAGING INTERVALS FOR SPECTRAL ANALYSIS OF NONSTATIONARY TURBULENCE

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Abstract. We formulate a method for determining the smallest time interval ΔT over which a turbulence time series can be averaged to decompose it into instantaneous *mean* and *random* components. From the *random* part the method defines the optimal interval (or averaging window) AW over which this part should be averaged to obtain the instantaneous spectrum. Both ΔT and AW vary randomly with time and depend on physical properties of the turbulence. ΔT also depends on the accuracy of the measurements and is thus independent of AW. Interesting features of the method are its real-time capability and the non-equality between AW and ΔT.

Keywords: Turbulence spectrum, Nonstationarity, Real-time, Averaging windows, Intermittency.

1. Introduction

The atmospheric boundary layer (ABL) is by nature nonstationary at all time scales. Another name for nonstationarity is *intermittency*, but, for the ABL, intermittency generally refers to short-term events. A key to understanding ABL intermittency is the ability to say something meaningful about the turbulence during such events.

Many researchers have reported results of investigations into requisite averaging intervals for estimating turbulence statistics (e.g., Bradshaw, 1971; Lumley and Panofsky, 1964, p. 36ff.; Gupta et al., 1971; Wyngaard, 1973; Blackwelder and Kaplan, 1976; Wyngaard and Clifford, 1978; Sreenivasan et al., 1978; Andreas, 1988; Kaimal et al., 1989; Lenschow et al., 1994; Gluhovsky and Agee, 1994; Gupta, 1996). None, however, has adequately considered the intervals necessary to determine turbulence statistics within intermittent ABL events.

Here we extend the idea of minimum averaging to its lowest limit, one that approaches the duration of intermittent events – tens of seconds. Keep in mind that spectral analysis of the ABL requires quantifying not only the instantaneous spectrum Φ but also changes in it from one intermittent event to the next. Such changes typically appear in frequency content, distribution, domain, and bandwidth.

To accomplish this, we formulate a method for identifying the smallest interval ΔT over which an ABL time series can be averaged to define its instantaneous

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mean and accompanying *deviations* from the mean. In some circles, *mean* is referred to as the *systematic* part and *deviations* are called the *random* part (see Bendat and Piersol, 1971). From the *random* part, the method determines the optimal interval (or window) AW over which this part should be averaged to estimate Φ.

The interesting feature of these determinations is not only that AW and ΔT are determined in real time and depend on instantaneous properties of the ABL, but also that AW does not always equal ΔT. The difference between AW and ΔT is due to dynamical effects and not to kinematical scale changes. That is, it is not possible to re-scale the turbulence to make AW and ΔT equal to one another. This difference is consistent with the fact that *mean* and *random* components represent different degrees of freedom. In determining Φ we treat the *mean* as an instantaneous DC component; from Φ emerges all the parameters necessary to assess the instantaneous character of ABL turbulence.

The method localizes Φ by assigning uniform weight to the *random* part of the time series within AW and zero weight to anything outside. The location of AW in time is designated by t, the *present* time. The length of AW is proportional to a t-dependent parameter called the *memory*. As t advances, the memory changes; and, thus, the magnitude of AW varies. This has the effect of varying the 'block length' of data used in computing Φ (cf. Henjes, 1997). The magnitude and variability of the block length here, though, are defined by the *random* part of the ABL time series rather than by the analyst. In this way, the method adapts to instantaneous ABL properties and reduces the likelihood that short-lived events are lost or masked due to user-defined averaging intervals that are too short or too long.

Its real-time capability makes our method more versatile than existing algorithms, such as the *sliding* FFT (Fast Fourier Transform) and *variable interval time averaging* (VITA) (Gupta et al., 1971; Blackwelder and Kaplan, 1976; Gupta, 1996). Real-time capability in *sliding* FFT and VITA is unachievable because (i) both use information from future ABL behaviour to determine present frequency information; and (ii) both require interactive user input, which cannot reliably emulate the changing properties of the ABL. Information from the future is used whenever an algorithm determines ABL behaviour at time t by using information at time $t + T$, $T > 0$, and all intervening times. The method formulated here uses only information from the present and not-too-distant past to define Φ; no information from the future is required. The method formulated here requires no interactive user input. Lastly, *sliding* FFT and VITA, in contrast to our method, use the same averaging interval for *mean* and *random* components of the turbulence.

For an optimal AW, the resulting Φ depends on t but not on AW. That is, in spite of inherent variability, AW is defined in such a way that Φ is strictly $\Phi(t, \omega)$, *not* $\Phi(t, \omega, \text{AW})$, where ω is radian frequency. As t advances, and successive sets of ΔT, AW, and Φ are identified, $\Phi(t, \omega)$ estimates the instantaneous spectrum of the ABL. We demonstrate our method by applying it to wind speed data collected in the ABL.

2. Formulation

2.1. Separating *mean* and *random* parts

Consider the time series $S(Z, \gamma)$, which is sampled for the period $[t-T, t]$, where t is the *present* time and $T > 0$ can be on the order of hours. γ is the 'running time', and $S(Z, \gamma)$ is sampled fast enough to contain inertial subrange information. Z is the height above the ground where the measurements were made and, for our analysis, is disposable.

$S(\gamma)$ is first averaged according to

$$\frac{1}{\Delta T}\int_{t-\Delta T}^{t} S(\gamma)\mathrm{d}\gamma = \bar{S}(t, \Delta T), \tag{1}$$

where $\Delta T < T$. Note the present time is the upper limit of this integration, and only information from the present and earlier are used. No information forward of t (i.e., the future) is invoked, making (1) a *real-time* average.

The intent is to determine how far back in time, defined by $\Delta T > 0$, to begin the integration in (1) such that $\bar{S}(t, \Delta T)$ estimates the ABL *mean* at time t. Although here we implement (1) in an off-line fashion, (1) can be hardwired into a signal processor to generate information that is as near real-time as possible for the particular processor. The requisite ΔT is itself t dependent; and for (1) to be a reliable measure of a local average, ΔT must be on the order of magnitude of the related time scale of the ABL. Once that ΔT is matched, though, $\bar{S}(t, \Delta T)$ loses its ΔT dependence and becomes simply $\bar{S}(t)$.

If the expected value of $S(\gamma)$ is $\langle S(\gamma)\rangle = \mu(\gamma)$ and $\mu(\gamma) \approx$ constant, say $\mu(t)$, over $[t - \Delta T, t]$, $\mu(t)$ is then a valid estimate of $\langle \bar{S}(t, \Delta T)\rangle$. For the ABL, the requisite constancy of $\mu(\gamma)$ can be enforced by removing any trend that may exist in $S(\gamma)$ over ΔT (cf. Treviño, 1985).

Using (1), we form the sequence of values $\bar{S}(\gamma, \Delta T)$ for γ in the domain $t - \Delta T \leq \gamma \leq t$. This requires values of $S(\gamma)$ as far back as $2\Delta T = H$ (H for 'history'). For example, if ΔT is 5 s, the sequence of $\bar{S}(\gamma, \Delta T)$ values formed in this way contains 5 s worth of 5 s averages, i.e., 10 s total of past and present information. The motivation for forming the sequence $\bar{S}(\gamma, \Delta T)$ in this way is to identify the *random* part of $S(\gamma)$ also over $t - \Delta T \leq \gamma \leq t$. The procedure for doing this is defined in the following paragraph. In this way, both $\bar{S}(\gamma, \Delta T)$ and the *random* part of $S(\gamma)$ can be identified over the same time scale (ΔT).

From the sequence $\bar{S}(\gamma, \Delta T)$, we form the time series

$$x(\gamma, \Delta T) = S(\gamma) - \bar{S}(\gamma, \Delta T). \tag{2}$$

Table I illustrates this procedure for a present time of 09:46 local time and averages of 1 s. Column 1 is actual time beginning at 09:45:57; column 2 is actual wind speed data recorded at 10 Hz; column 3 is the 1 s averages (10 data points) of the data in column 2, beginning at 09:46 (the present) and running backwards until 09:45:59.1. That is, all the elements in column 3 are averages defined as in (1) for the succession of 'present' times 09:45:59.1, 09:45:59.2, 09:45:59.3, …, 09:46:00. The quantity in column 3 corresponding to 09:45:59.1 is the average of the ten quantities in Column 2 from 09:45:59.1 back to 09:45:58.2.

Column 4 is the difference between column 2 and column 3. If 1 s was indeed the window that 'matches' the related time scale of the data, Column 3 would then be the time-dependent *mean* of the data for the succession of 'present' times 09:45:59.1, 09:45:59.2, 09:45:59.3, …, 09:46:00, and column 4 would be the corresponding time-dependent *random* part. An alternate designation for the totality of information in Column 4 is $\chi(\gamma)$, $t - \Delta T \leq \gamma \leq t$. This notation will be used hereafter.

Next, evaluate the quantity

$$\text{m.s.e.} = \left\{ \frac{1}{\Delta T} \int_{t-\Delta T}^{t} \chi(\gamma) \mathrm{d}\gamma \right\}^2$$

$$= \left\{ \left(\frac{1}{\Delta T} \right)^2 \int_{t-\Delta T}^{t} \int_{t-\Delta T}^{t} \chi(\gamma_1)\chi(\gamma_2) \mathrm{d}\gamma_1 \mathrm{d}\gamma_2 \right\} = \bar{\chi}^2(t, \Delta T) \qquad (3)$$

for a specified t (e.g., 09:46) and values of ΔT beginning, say, with ΔT in (1) initially the inverse of the sampling rate (SR) and then increasing by successive doubling. Thus, if SR is 10 Hz, the first ΔT is 0.1 s, and successive values are 0.2 s, 0.4 s, 0.8 s, …. Care must be taken, though, to ensure that H does not exceed T. If it does, select a later t and begin again.

In (3), m.s.e. designates the *mean-square error* (see Papoulis, 1965, p. 323, et seq.), and $\chi(\gamma)$ is a time series whose average is not identically zero, either in the time-average sense (over the requisite ΔT) or in the ensemble-average sense – cf., Column 4, Table I. Both averages, however, are 'small', with the definition of 'small' to be given shortly.

With the transformation of coordinates $\gamma = (\gamma_1 + \gamma_2)/2$ and $\tau = \gamma_2 - \gamma_1$, the double integral within { } in (3) becomes

$$\text{m.s.e.} = \frac{1}{\Delta T} \int_{-\Delta T}^{\Delta T} \left(\left(\frac{1}{\Delta T - |\tau|} \right) \int_{t-\Delta T+|\tau|/2}^{t-|\tau|/2} \chi\left(\gamma - \frac{\tau}{2}\right) \chi\left(\gamma + \frac{\tau}{2}\right) \mathrm{d}\gamma \right) \mathrm{d}\tau, \qquad (4)$$

where both the scale factor $(\Delta T - |\tau|)^{-1}$ and the limits of the integration in γ are adjusted to accommodate the fact that only a finite amount of data are available

TABLE I

Example of obtaining the *random* part of a series of turbulence data using (1) and (2). Here $\Delta T = 1$ s.

Actual time $[\gamma]$	Measured data $[S(\gamma)]$	First average $[\bar{S}(\gamma, \Delta T)]$	Random part $[\chi(\gamma)]$
09:45:57.0	1.79		
09:45:57.1	1.78		
09:45:57.2	1.76		
09:45:57.3	1.71		
09:45:57.4	1.63		
09:45:57.5	1.55		
09:45:57.6	1.44		
09:45:57.7	1.52		
09:45:57.8	1.42		
09:45:57.9	1.53		
09:45:58.0	1.58		
09:45:58.1	1.59		
09:45:58.2	1.48		
09:45:58.3	1.60		
09:45:58.4	1.66		
09:45:58.5	1.66		
09:45:58.6	1.68		
09:45:58.7	1.74		
09:45:58.8	1.70		
09:45:58.9	1.84		
09:45:59.0	1.82		
09:45:59.1	1.77	1.70	0.07
09:45:59.2	1.72	1.72	0.00
09:45:59.3	1.65	1.72	−0.07
09:45:59.4	1.65	1.72	−0.07
09:45:59.5	1.78	1.74	0.04
09:45:59.6	1.71	1.74	−0.03
09:45:59.7	1.65	1.73	−0.08
09:45:59.8	1.64	1.72	−0.08
09:45:59.9	1.74	1.71	0.03
09:46:00.0	1.86	1.72	0.14

the computation. The expression within the outer ( ) is the autocorrelation. The τ-dependence of the scale factor can be eliminated if ΔT is considerably larger than the largest value of τ for which the autocorrelation is non-zero; that is, if ΔT is large enough that $(\Delta T - |\tau|)^{-1} \approx (\Delta T)^{-1}$ for the largest meaningful value of τ in the autocorrelation. We show later that this relationship between ΔT and τ will always be the case. The resulting autocorrelation then depends on t, ΔT, and τ but is subsequently integrated over τ. Thus, for a nonstationary ABL, the real-time dependence of m.s.e. is never lost.

For large ΔT, though, the integral within ( ) in (4) stabilizes at $C(t, \tau)$. The stability is such that the subsequent integration in (4), viz. $\int_{-\Delta T}^{\Delta T} C(t, \tau)\ \mathrm{d}\tau$, is unchanged by further increasing ΔT; that is, this stability eliminates the integral's dependence on ΔT. Thus, by the 'requisite ΔT' above we mean a ΔT long enough that

$$\frac{1}{\Delta T} \int_{-\Delta T}^{\Delta T} C(t, \tau) \mathrm{d}\tau \approx 0.$$

As a result, $\bar{S}(t, \Delta T)$ is a valid estimate of the *mean* at time t, and $\chi(\gamma)$, $t - \Delta T \le \gamma \le t$, represents *deviations* from the *mean*. The right-hand side of (4) is then $\sigma^2(t)\Lambda(t)/\Delta T$, where $\sigma^2(t)$ is the *variance* of $\chi(\gamma)$, $t - \Delta T \le \gamma \le t$, and $\Lambda(t)$ is its *integral* scale. Because of the 'requisite' nature of ΔT, σ^2 and Λ do not depend on ΔT.

In numerical work, though, m.s.e. is taken to be zero whenever it becomes smaller than the square of the accuracy (ACC) of the measurements. This is equivalent to saying that the absolute value of $\bar{\chi}(t, \Delta T)$ in (3) is less than the absolute value of ACC, which effectively makes $\bar{\chi}(t, \Delta T)$ equal zero to within the accuracy of the measurements. It also makes the expected value of $\chi(\gamma)$ equal zero to within the same accuracy. This is what we mean above by 'small', viz. $\bar{\chi}(t, \Delta T)$ is not identically zero but is smaller than what can be detected with our sampling device.

The smallest ΔT that yields this condition is then the value for which, (i) a viable estimate of the *mean* at time t can be defined, and (ii) a valid time description of the related *random* part can be determined. For example, for the anemometer we use to measure wind speed, the accuracy of the measurements is specified by the manufacturer as $\pm 1\%$ of the wind speed. This we take to mean that at any instant, ACC = $0.01\bar{S}(t)$. The magnitude of ΔT for our analysis is therefore defined in the m.s.e. sense through $(\sigma^2\Lambda/\Delta T) < 10^{-4}\bar{S}^2$. Andreas and Treviño (1997) also use instrument accuracy as a criterion for deciding how to proceed with a turbulence analysis.

2.2. Determining the Spectrum

We determine the magnitude of the averaging window AW from the *memory* of the turbulence, denoted as L and defined by

$$L(t) = \max \left\{ \frac{\left[\int_{t-\beta}^{t} \chi(\gamma)\mathrm{d}\gamma\right]^2}{2\int_{t-\beta}^{t} \chi^2(\gamma)\mathrm{d}\gamma} \right\}. \tag{5}$$

Here β is allowed to vary from a lower limit of zero to its upper limit ΔT; recall that $\chi(\gamma)$ is defined over $t - \Delta T \leq \gamma \leq t$. Note that L is a parameter that has dimensions of time and is, by definition, a positive number; also note that L, because it is defined as a *maximum*, is not a function of β. A more recognizable form for L is (cf. (3) and (4))

$$\begin{aligned} L(t) &= \max \left\{ \frac{\int_{-\beta}^{\beta} \left(\int_{t-\beta+|\tau|/2}^{t-|\tau|/2} \chi\left(\gamma - \frac{\tau}{2}\right) \chi\left(\gamma + \frac{\tau}{2}\right) \mathrm{d}\gamma \right) \mathrm{d}\tau}{2\int_{t-\beta}^{t} \chi^2(\gamma)\mathrm{d}\gamma} \right\} \\ &= \max \left\{ \int_{-\beta}^{\beta} \hat{R}(t, \tau, \beta)\mathrm{d}\tau \right\}, \end{aligned} \tag{6}$$

where $\hat{R}$ is a β-dependent approximation to the normalized autocorrelation of $\chi(\gamma), t - \Delta T \leq \gamma \leq t$.

As β varies from zero to ΔT, L equals Λ in those cases when $\Delta T \gg \Lambda$ *and the autocorrelation* of $\chi(\gamma)$ *is greater than or equal to zero for all values of* τ. For our data, the condition $\Delta T \gg \Lambda$ is satisfied when $(\sigma/\bar{S}) > 0.03$. In those cases when $\Delta T \gg \Lambda$ and the autocorrelation becomes negative for some values of τ, L is greater than Λ. In the cases when $(\sigma/\bar{S}) < 0.03$, L can still be determined, but the measured signal is highly concentrated (small deviations) about its *mean*.

In all cases, though, (6) is a measure of the separation in time beyond which variables cease to be correlated. It is thus a measure of the *degree of randomness*. Small L corresponds to a highly random (short memory) ABL; and large L, to not-so-random. Since classical analysis methods do not incorporate effects of such variability, they can be designated as time-*invariant* memory methods (TIMMs). The method formulated here, conversely, is designated the time-*dependent* memory method (TDMM). We show later, though, that TDMM reduces to TIMM when the ABL is stationary. No TIMM, though, can produce the same information as does TDMM when the ABL is nonstationary. VITA, on the other hand, also incorporates effects of averaging window variability, but its averaging window is defined by the user.

In view of this interpretation, then, the magnitude of AW is taken to be 10L. Since L is always greater than or equal to Λ, it follows that AW $\geq 10\Lambda$. This ensures that, (i) a representative sample of the *random* part is analyzed, and (ii) all correlations are lost within AW. As indicated in (7) below, defining AW this way assigns uniform weight to values of $\chi(\gamma)$ that are backward from t by an amount less than or equal to 10L and assigns no weight to values of $\chi(\gamma)$ backward from t by more than 10L. It also assigns no weight to elements of the ABL *forward* in time from t. This is compatible with the fact that, in physical phenomena, the future cannot influence the present.

Defining AW this way requires that $\chi(\gamma)$ be stationary only over $[t - \text{AW}, t]$ and not over $[t - \Delta T, t]$. As t increases to define successive sets of ΔT, $\bar{S}(t)$, and AW, the ABL is approximated by a succession of (locally) stationary segments (cf. Holman and Hart, 1972) which, if necessary, can overlap one another. The time extent of the segments here, though, are different for the *mean* and *random* components. This reflects the adaptiveness of TDMM and is consistent with the premise that the best averaging window 'depends on the signal, and may differ for different components' (Jones and Parks, 1990). Typically, AW = 10L will be less than ΔT; but if this is not the case, ΔT should be taken as AW since the choice of the numerical value of 10 in 10L is a decision made by the analyst. Some investigators (see Henjes, 1997) prefer to use windows smaller than 10L.

Determining the spectrum of $\chi(\gamma)$ begins with

$$X(t, \omega, \text{AW}) = \frac{1}{\sqrt{\text{AW}}} \int_{t-\text{AW}}^{t} \chi(\gamma) \mathrm{e}^{-\iota\omega(\gamma - t)} \mathrm{d}\gamma, \tag{7}$$

where ω is in the interval $[2\pi/\text{AW}, \pi(\text{SR})]$. Note (7) is a variant of the short-time Fourier transform (STFT), which is the accepted method of tracking frequency information as it changes with time (Cohen, 1995). Further note that (7) is also a real-time average. The peculiar feature of (7), though, is that its 'shortness' is defined by a t-dependent property of $\chi(\gamma)$, $t - \Delta T \leq \gamma \leq t$, which, aside from the factor of 10, *cannot be adjusted by the user*. This is what ensures that we are using AW to analyze $\chi(\gamma)$, and not vice versa. Specifically, that the determined properties of $X(t, \omega, \text{AW})$ are more closely related to $\chi(\gamma)$ as opposed to AW; the mathematics of Fourier analysis makes no distinction (cf. Blackwelder and Kaplan, 1976).

Even though STFT is known to introduce high-frequency information associated with the boxcar averaging interval, we show below, see Equation (11), that these effects are negligible when determining Φ. That is, even though TDMM produces spurious information in the STFT of $\chi(\gamma)$, this information does not carry over into Φ. The reason is that, while (7) *abruptly* truncates $\chi(\gamma)$ at both t and $t -$ AW, producing 'end effects', the function Φ nonetheless decays *gradually* to zero over an interval of sufficient extent to make these effects negligible. Note that X is a function of AW, the reason being that even though AW = 10L is considered long enough to ensure all relevant information is included in Φ, the STFT of $\chi(\gamma)$ still depends on AW.

As in (3) and (4) above, the product

$$|X(t, \omega, \mathrm{AW})|^2 = \frac{1}{\mathrm{AW}} \int_{t-\mathrm{AW}}^{t} \int_{t-\mathrm{AW}}^{t} \chi(\gamma_1)\chi(\gamma_2)\mathrm{e}^{-\iota\omega(\gamma_2-\gamma_1)}\mathrm{d}\gamma_1\mathrm{d}\gamma_2 \tag{8}$$

becomes

$$|X(t, \omega, \mathrm{AW})|^2 = \int_{-\mathrm{AW}}^{\mathrm{AW}} C(t, \tau)\mathrm{e}^{-\iota\omega\tau}\mathrm{d}\tau \approx \int_{-\infty}^{\infty} C(t, \tau)\mathrm{e}^{-\iota\omega\tau}\mathrm{d}\tau = \Phi(t, \omega). \tag{9}$$

Note that (9) begins on the left as a function of three variables and ends up on the right as a function of two. The first part of that reduction is

$$\frac{1}{\mathrm{AW}} \int_{t-\mathrm{AW}+|\tau|/2}^{t-|\tau|/2} \chi\left(\gamma - \frac{\tau}{2}\right) \chi\left(\gamma + \frac{\tau}{2}\right) \mathrm{d}\gamma = C(t, \tau). \tag{10}$$

This is justified because, even though AW is (presumably) less than ΔT, it is by definition still large enough to include all relevant information in $C(t, \tau)$. The right-hand side of (10) is thus independent of AW.

The last reduction

$$\int_{-\mathrm{AW}}^{\mathrm{AW}} C(t, \tau)\mathrm{e}^{-\iota\omega\tau}\mathrm{d}\tau \approx \int_{-\infty}^{\infty} C(t, \tau)\mathrm{e}^{-\iota\omega\tau}\mathrm{d}\tau \tag{11}$$

approximates the true spectrum by its truncated counterpart. This is justified because there is no information in $C(t, \tau)$ beyond τ = 10L that contributes to the value of the integral. In short, Fourier transforming $C(t, \tau)$ in τ over the signal-defined interval $[-\mathrm{AW}, \mathrm{AW}]$ nullifies the effect of windowing $\chi(\gamma)$ over $[t - \mathrm{AW}, t]$. This reduction is important for computing Φ since it is now not necessary to allow $\mathrm{AW} \to \infty$ to make that computation.

The spectral nature of Φ follows from the inverse of (9); specifically,

$$\sigma^2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(t, \omega)\mathrm{d}\omega \approx \frac{1}{\pi} \int_{\omega_s}^{\omega_1} \Phi(t, \omega)\mathrm{d}\omega, \tag{12}$$

where $\omega_s = 2\pi/\mathrm{AW}$ is the smallest frequency resolvable from $\chi(\gamma)$, and ω_l is the Nyquist frequency (= π(SR)). Note there are two t-dependencies on the right-hand side of (12); one in Φ and the other in ω_s. A meaningful analysis must identify both.

A note of caution is due here. It would seem to the unsuspecting that once AW is defined, the FFT algorithm, or some variation of it, could be invoked to determine frequency information in the *random* part of the turbulence defined over $[t - \mathrm{AW},$

t]. This is not necessarily the case since there is no guarantee that AW will be defined by the turbulence to be a power of 2.

3. Illustrative Examples

We applied TDMM to longitudinal wind speed $U(\gamma)$ measured with an ATI (Applied Technologies, Inc., Boulder, CO) three-axis sonic anemometer positioned 4 m above the ground and digitized at 10 Hz. We selected data recorded on 4 August 1991 at the Sevilletta National Wildlife Refuge in New Mexico beginning at 20:00 local time. Andreas et al. (1998) describe the measurements in more detail. According to ATI, the accuracy of the data is $\pm$ 1% of the *mean* wind.

We took data beginning at 20:06 and first detrended them according to the following procedure. We assumed that over a data block of length $[t - \Delta T, t]$ the wind had a *mean* defined as $\mu(\gamma) \approx \mu_1\gamma + \mu_0$, where μ_1 and μ_0 are constants for each block but can vary from block to block. That is, we allowed for the possibility that there may be a time-dependent trend in the wind.

The value of μ_1 was then approximated from the data by the random variable

$$\boldsymbol{\mu}_1(t, \Delta T, \Delta\gamma) = \frac{1}{\Delta T}\int_{t-\Delta T}^{t} \frac{U(\gamma + \Delta\gamma) - U(\gamma)}{\Delta\gamma}\mathrm{d}\gamma, \tag{13}$$

where $\Delta\gamma$ = (SR)$^{-1}$. Note that, for the assumed time-dependence of $\mu(\gamma)$, the expected value of (13) is

$$\langle\boldsymbol{\mu}_1(t, \Delta T, \Delta\gamma)\rangle = \frac{1}{\Delta T}\int_{t-\Delta T}^{t} \frac{[\mu_1(\gamma + \Delta\gamma) + \mu_0] - [\mu_1\gamma + \mu_0]}{\Delta\gamma}\mathrm{d}\gamma = \mu_1. \tag{14}$$

That is, (13) is an *unbiased* estimator of the instantaneous slope of $\mu(\gamma)$ (cf. Treviño, 1985). This value was then multiplied by γ and subtracted from $U(\gamma)$ over the block in question. The remainder of that subtraction is the time series $u(\gamma) = U(\gamma) - \mu_1\gamma$, which has a *mean* that is approximately constant over $[t - \Delta T, t]$, as required by (1).

For the 20:06 data block, the algorithm yielded μ_1 = 0.01 m s^{-2}, indicating a mild trend in the wind. The requisite length of the block was found to be ΔT = 25.6 s, which required that H = 51.2 s (recall that data are available as far back as 20:00). For this 25.6 s data block, we found L = 1.46 s and, thus, AW = 14.6 s. The minimum value of the set of averages defined by (1) for this block was 1.61 m s^{-1} and its maximum was 1.74 m s^{-1}. This we took as compatible with the condition that the *mean* of the detrended series (μ_0) is approximately constant.

We also computed another scale for the data in question,

$$\eta(t) = \left(\frac{\int_{t-\mathrm{AW}}^{t} u^2(\gamma)\mathrm{d}\gamma}{\int_{t-\mathrm{AW}}^{t} \left(\frac{\mathrm{d}u(\gamma)}{\mathrm{d}\gamma}\right)^2 \mathrm{d}\gamma} \right)^{0.5}, \tag{15}$$

where we approximated $\mathrm{d}u(\gamma)/\mathrm{d}\gamma$ as (SR)$[u(\gamma_{i+1}) - u(\gamma_i)]$. Note that $\eta(t)$ is by definition also a positive number. It is characteristic of the small-scale behaviour of the ABL; and, even though it too is a time scale, *it is not proportional to* L. Specifically, L can be thought of as a *macroscale* and η as a *microscale*. η is the equivalent of the Taylor microscale λ according to $\eta = \lambda/\bar{U}$ when SR is high enough ($>$1000 Hz) and the sampling volume of the sensor is small enough.

The difference between L and η is that η is sensitive to *local* rearrangement of the data points while L is not. In other words, if we take the given data and display them versus time in a column on a spreadsheet and then (locally) scramble this column using a SORT command, η will likely change by a noticeable amount. L, on the other hand, because it is defined as a *maximum*, will not. The value of β, though, where this maximum occurs will change.

The value of η we found for 20:06 is 0.25 s; our results are indicated in Figure 1. The function $\omega\Phi$(20:06, ω) is plotted versus ω in rad s^{-1} (solid line). The broken line has $-2/3$ slope, and σ(20:06) = 0.27 m s^{-1}. The fact that the $-2/3$ slope line does not pass squarely through the data indicates the inertial subrange is (slightly) contaminated by large-scale anisotropy.

We next performed the analysis on a data segment 10 min into the record (20:10). The value of μ_1 there was 0.00 m s^{-2} indicating no trend. The value of μ_0 oscillated between 2.23 m s^{-1} and 2.30 m s^{-1}, and ΔT was 51.2 s (H = 102.4 s). For these data, though, L was 1.08 s, which gave AW = 10.8 s and values of σ and η equal to 0.21 m s^{-1} and 0.18 s, respectively. The related $\omega\Phi$(20:10, ω) is shown in Figure 2.

Repeating this procedure beginning now at, say, t = 20:12 (or some later time), will yield values for σ(20:12) and L(20:12). These iterations will produce a sequence of values for σ, L, and η, as well as functions $\omega\Phi(\gamma, \omega)$. If these collectively do not change with running time, we can conclude that the ABL is stationary. In this case, $\omega\Phi(\gamma, \omega)$ will be similar to a spectrum obtained using FFT. If the values do change, however, the ABL is nonstationary, and the time scale over which they change is a measure of the time scale of the phenomenon forcing the changes.

A unified method for assessing the degree-of-nonstationarity in a random signal is found in Treviño (1982). It is called the *generalized frequency spectrum* (GFS) concept. The essence of the GFS concept is to carry out a second frequency analysis of $\Phi(\gamma, \omega)$ to obtain the GFS, denoted $\Phi(\Omega, \omega)$. The 'spread' of $\Phi(\Omega, \omega)$ about Ω = 0 is then a measure of the degree-of-nonstationarity. Since $\Phi(\Omega, \omega)$ is in

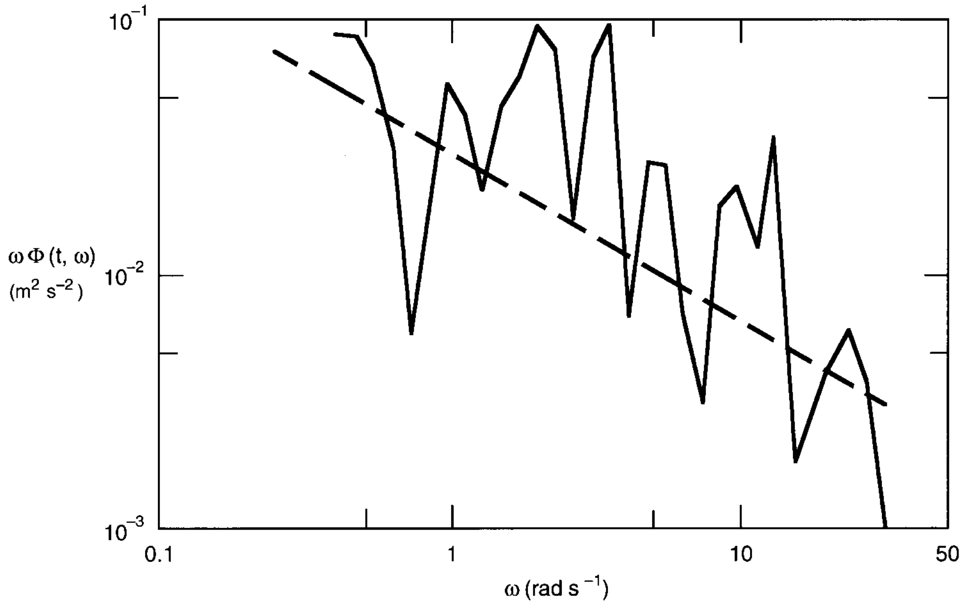

Figure 1. The function $\omega\Phi(20{:}06, \omega)$ versus ω in rad s^{-1} (solid line). The values of η and L at this time are 0.25 s and 1.46 s (AW = 14.6 s), respectively. The value of ΔT is 25.6 s. The broken line has $-2/3$ slope, and σ = 0.27 m s^{-1} is determined by averaging $\chi^2(\gamma)$ over [20:05:45.4, 20:06].

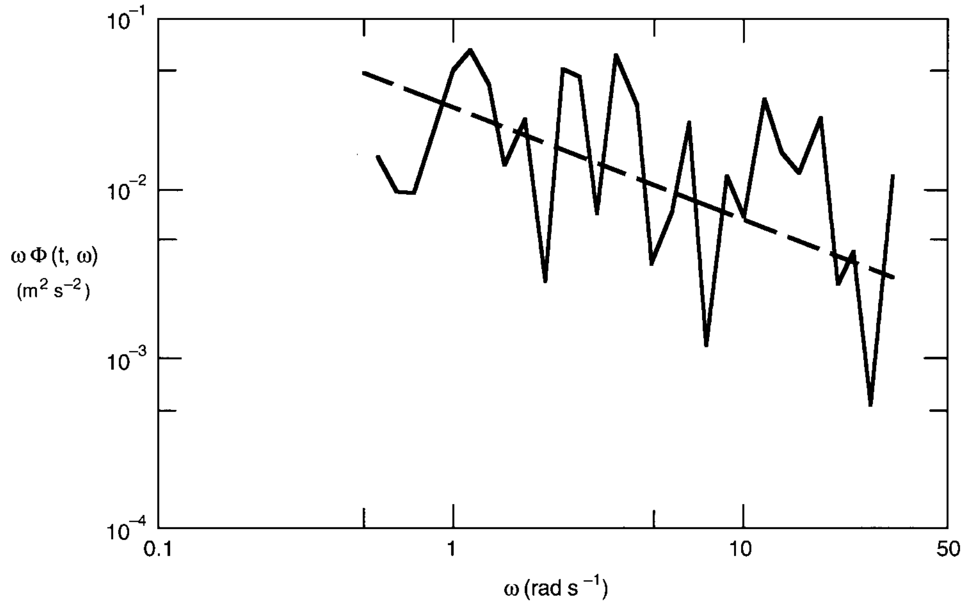

Figure 2. The function $\omega\Phi(20{:}10, \omega)$ versus ω in rad s^{-1} (solid line). The values of η and L at this time are 0.18 s and 1.08 s (AW =10.8 s), respectively. The value of ΔT is 51.2 s. The broken line has $-2/3$ slope, and σ = 0.21 m s^{-1} is determined by averaging $\chi^2(\gamma)$ over [20:09:49.2, 20:10].

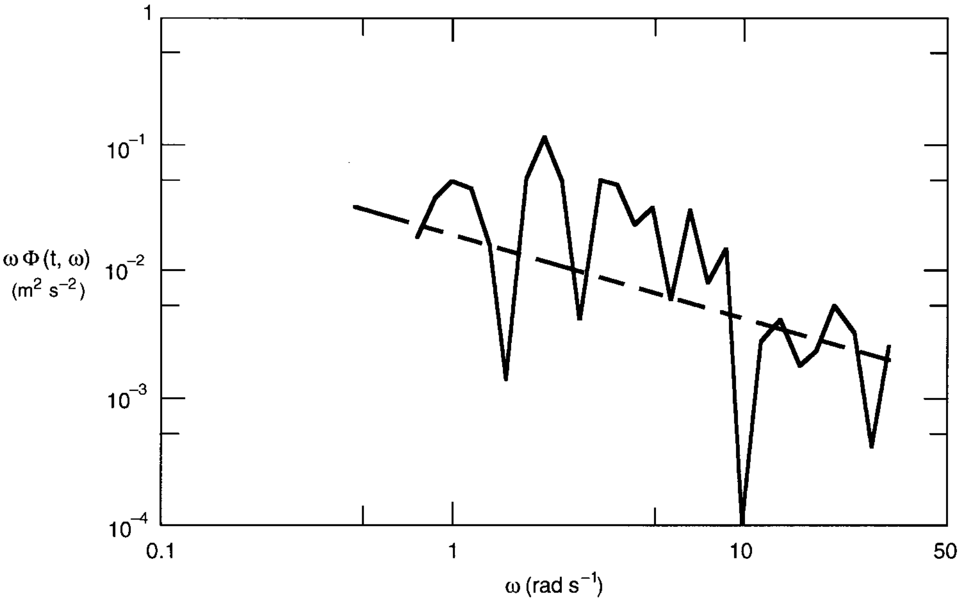

Figure 3. The function $\omega\Phi$(20:14, ω) versus ω in rad s^{-1} (solid line). The values of η and L are 0.22 s and 0.88 s (AW = 8.8 s), respectively. The value of ΔT is 51.2 s. The broken line has $-2/3$ slope, and σ = 0.19 m s^{-1} is determined by averaging $\chi^2(\gamma)$ over [20:13:51.2, 20:14].

general complex (i.e., it has real and imaginary parts), it is best to analyze the behaviour of $|\Phi(\Omega, \omega)|$ about Ω = 0.

The time scale of changes in σ, we should note, is not the same as the time scale of changes in L (or η). That is, it is possible for σ to vary slowly and for L to vary quickly (or vice versa). Also keep in mind that instants in time when σ is 'large' and L is 'small' correspond to energetic, highly random events. Conversely, instants in time when σ is 'small' and L is 'large' correspond to quiet, less random events.

As further examples of TDMM, we also performed the analysis on a segment 14 min into the record. The value of μ_1 there was again 0.00 m s^{-2}. The value of μ_0 oscillated between 1.52 m s^{-1} and 1.63 m s^{-1}, and ΔT was again 51.2 s (H = 102.4 s). This time L was 0.88 s, which gave AW = 8.8 s and values of σ and η equal to 0.19 m s $^{-1}$ and 0.22 s, respectively. The related $\omega\Phi$(20:14, ω) is shown in Figure 3.

We took yet another segment of the series, this time 18 min into the record. The value of μ_1 there was again 0.00 m s^{-2}, and the value of μ_0 oscillated between 1.85 m s^{-1} and 1.92 m s^{-1}. The ΔT was 51.2 s (H = 102.4 s) and L was 3.04 s; values of σ and η were 0.23 m s^{-1} and 0.28 s, respectively. The function $\omega\Phi$(20:18, ω) is shown in Figure 4.

In Table II, we summarize the statistics that our four analyses have produced. From μ_0, L, and σ especially, we see that this time series is moderately nonstation-

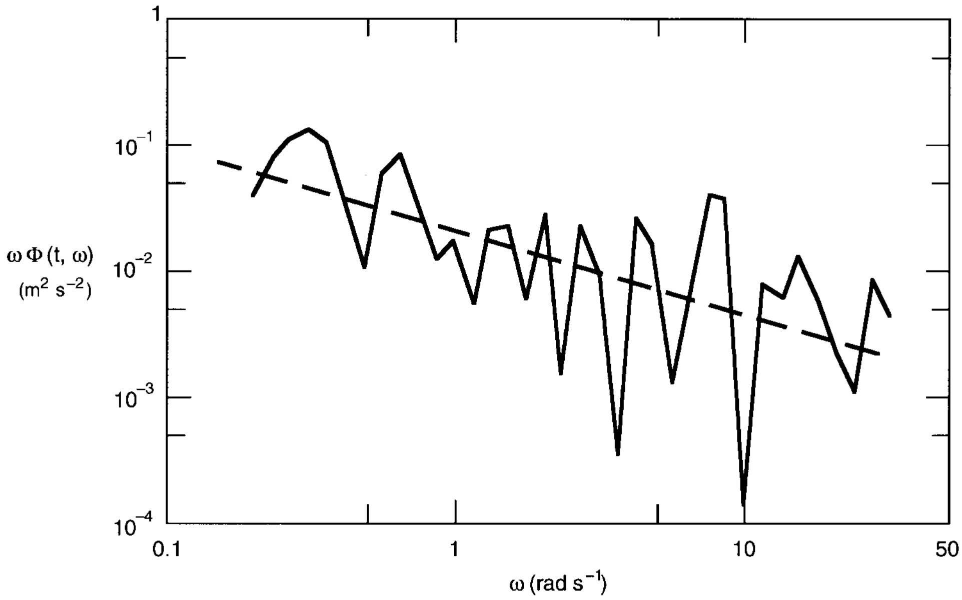

Figure 4. The function $\omega\Phi(20{:}18,\ \omega)$ versus ω in rad s^{-1} (solid line). The values of η and L are 0.28 s and 3.04 s (AW = 30.4 s), respectively. The value of ΔT is 51.2 s. The broken line has $-2/3$ slope, and σ = 0.23 m s^{-1} is determined by averaging $\chi^2(\gamma)$ over [20:17:29.6, 20:18].

TABLE II

Statistics derived by our time-dependent memory method (TDMM) for the data segments that yielded Figures 1–4.

Time	μ_1 (m s^{-2})	μ_0 Range (m s^{-1})	ΔT (s)	L (s)	AW = 10L (s)	η (s)	σ (m s^{-1})
20:06	0.01	1.61–1.74	25.6	1.46	14.6	0.21	0.27
20:10	0.00	2.23–2.30	51.2	1.08	10.8	0.18	0.21
20:14	0.00	1.52–1.63	51.2	0.88	8.8	0.22	0.19
20:18	0.00	1.85–1.92	51.2	3.04	30.4	0.28	0.23

ary. In practice, we would prepare more closely spaced samples of the statistics summarized in Table II and look for patterns in the nonstationarity. For example, intense mixing events would yield periods with smaller L and larger σ, while more quiescent periods would show larger L and smaller σ.

4. Summary Remarks

Advancing our understanding of the ABL requires obtaining meaningful statistics that quantify intermittency. We have described an algorithm (TDMM) capable of extracting, in real-time, such statistics from events lasting only a few tens of seconds. TDMM makes no assumptions about the ABL other than it has a finite memory. In other words, correlations eventually go to zero. The event itself dictates the actual length of the required averaging. The quantities we compute are the variance, memory, a scale considerably shorter than the memory akin to the Taylor microscale, and the spectrum. The variations in these with time characterize intermittency. We summarize TDMM by delineating the sequence of computations that produce the spectral result and also the advantages that TDMM has over traditional spectral analysis algorithms.

An $H = 2\Delta T$ history of $S(\gamma)$ is required to ascertain the instantaneous *mean* $\bar{S}(t)$ and the *random* part $\chi(\gamma)$, $t - \Delta T \leq \gamma \leq t$; see (1) and (2) and recall that t represents the present time. Comparing m.s.e. with the sensor accuracy defines ΔT, see Equation (3). The memory L is then found from $\chi(\gamma)$, see (5) and (6). Based on the physical interpretation of L, AW is then defined as 10L and used to obtain $\Phi(t, \omega)$, see Equation (7), et seq.

The method formulated here (i) is frequency-shift invariant, meaning that it is independent of the frequency domain of $\chi(\gamma)$; (ii) eliminates effects of user-specific biases such as user-selected window lengths; (iii) eliminates corruption caused by using a static algorithm (such as FFT) to analyse dynamic phenomena; (iv) avoids degradation caused by using information from the future; (v) identifies, in real time, *if* and *how* frequency content, distribution, domain, and bandwidth, are changing; and (vi) reduces the possibility that short-lived information is lost or masked due to uncharacteristically long (or short) averaging intervals.

The three parameters σ, η, and L, obtained serve to quantify the local structure of the ABL. A fourth parameter, the t-dependent *mean*, is also determined but, since it is removed from S, does not affect the behaviour of Φ. This parameter can be thought of as an instantaneous DC component. The exact relationship between ΔT and AW is known only a posteriori. Specifically, $(\Delta T/\text{AW}) > 0.1(\sigma/\text{ACC})^2$, where (recall) that ACC is $0.01\bar{S}$ for our data.

As a final point, turbulence researchers agree that the proper choice of scale is important in the search for Reynolds-number-independent features. Yet they typically use the same time interval for Reynolds averaging of all correlations being examined. The fact that classical Reynolds averaging has been used to investigate the structure of higher-order correlations and has not yet yielded an appreciable understanding of the turbulence problem might be due to an inappropriate choice of averaging times. The method formulated here addresses this concern.

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