On the Mean Meridional Mass Motions of the Stratosphere and Mesosphere

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ABSTRACT

Using a simplified, approximate “Lagrangian-mean” dynamical formulation, the mean meridional mass circulation of the stratosphere and mesosphere is discussed. Under solstice conditions, it is shown that this Lagrangian-mean circulation may be inferred, as a first approximation, from the Eulerian-mean diabatic heating. Diabatic heating rates for the solstices, originally derived by Murgatroyd and Goody (1958), result in Lagrangian-mean rising motion at the tropical tropopause, subsidence across the extra-tropical tropopause, and a very strong summer-to-winter pole flow in the mesosphere. This circulation is exactly that obtained by Murgatroyd and Singleton (1961) for the solstices. Those authors, however, attempted to identify this circulation as the Eulerian-mean motion, and were later criticized for their neglect of the meridional eddy heat flux in the calculation, which proved to be extremely important in the winter hemisphere. The present study, nevertheless, indicates that Murgatroyd and Singleton’s circulation may in fact be representative of actual air parcel motions in the stratosphere and mesosphere.

1. Introduction

Over the last two decades the stratosphere and mesosphere have come under close scrutiny with a greatly increased number of observations, particularly those brought about by the development of satellite remote sensing. These regions of the atmosphere have not received attention merely for the many intriguing dynamical phenomena found there, but also because of the presence of a host of chemical species whose interaction and transport are of importance to biological life on the earth’s surface. In particular, two such constituents, ozone and radioactive isotopes, have stimulated intense interest in the subject of atmospheric mass transport, so that both short-term and long-term effects of anthropogenic perturbations on these species may be accurately described.

Historically, these “conservative tracers” have been used to deduce the sense of the mass circulation of the upper atmosphere (e.g., Newell, 1962, and references therein). Calculation of these motions based on dynamical mechanisms per se was first attempted by Murgatroyd and Singleton (1961). Those authors used the radiative heating rates of Murgatroyd and Goody (1958) to deduce a mean meridional circulation consisting of rising motion at the tropical tropopause and subsidence across the extra-tropical tropopause, with a strong summer-to-winter pole flow in the mesosphere. Murgatroyd and Singleton felt that this circulation was quite consistent with previous observations of tracer transport. However, they suggested that because of discrepancies in the angular momentum budget of this circulation, eddy transport processes, which they had neglected, were probably important.

Leovy (1964) attempted to account for such eddies by parameterizing them in terms of a linear damping of the mean zonal wind and temperature. With this parameterization, Leovy was able to derive a mean meridional circulation reduced in magnitude from Murgatroyd and Singleton’s, whose associated Coriolis torques gave rise to a zonal wind field much like the one observed. It is to be noted, however, that the structure of Leovy’s mean meridional circulation was virtually unchanged from that of Murgatroyd and Singleton.

Observational studies, on the other hand, led several authors (Reed et al., 1963; Miyakoda, 1963; Julian and Labitzke, 1965; Murakami, 1965; Perry, 1967; Vincent, 1968) to conclude that in the winter stratosphere, these eddies are able to influence not only the magnitude, but also the structure of the mean meridional circulation. These authors found a thermally indirect cell to be forced by these waves, such that there exists downward motion in midlatitudes, accompanied by rising motion over the winter pole. Recently Adler (1975) and Hartmann (1976) have discovered a similar indirect cell in the Southern Hemisphere winter stratosphere.

Two conclusions, then, may be inferred from the above comments. First, deduction of the true Eulerian-mean meridional circulation requires a knowledge of the eddy fluxes of heat and momentum. In the winter stratosphere, these fluxes are generally as large as the other terms in the balance equations for these quantities.
(e.g., Newell, 1963). Second, however, it appears from Murgatroyd and Singleton (1961) that the deduction of the qualitative sense of mass transport, in fact, may not require a knowledge of these fluxes. This is suggested, for example, by the qualitative agreement of Murgatroyd and Singleton's circulation (in the calculation of which the eddy heat flux was ignored) with the familiar "Brewer–Dobson" cell (Brewer, 1949; Dobson, 1956).

These conclusions appear at first sight to be paradoxical. Nevertheless, in the present paper it will be shown that both of these conclusions are in harmony, and are quite reasonable. Demonstration of this rests on a knowledge of the well-known "non-interaction theorem" (Charney and Drazin, 1961; Eliassen and Palm, 1961; Dickinson, 1969; Holton, 1974; Andrews and McIntyre, 1976, 1978a,b; Boyd, 1976; Holton and Dunkerton, 1978). This theorem, among other things, describes precisely how eddies are able to force mean meridional circulations in situations where the mean flow is in thermal wind balance (Eliassen, 1950; McIntyre, 1977). More specifically, this theorem states that, to second-order in wave amplitude, waves which are "steady" (not locally growing in time) and "conservative" (not being internally forced or dissipated in any way) induce, under suitable boundary conditions, a mean meridional cell which acts to identically cancel the eddy fluxes of the waves themselves. Consequently, in such a case the waves do not give rise to changes in the basic state flow—hence the designation "non-interaction theorem".

The relevance of the non-interaction theorem to stratospheric dynamics has been lucidly demonstrated in a number of numerical models (e.g., Hunt and Manabe, 1968; Cunnold et al., 1975; Holton, 1976; Mahlman and Moxim, 1978). The role of eddies in forcing a compensating mean meridional circulation is an outstanding feature of these models. These studies have made it quite clear that we may not regard the extratropical mean meridional circulation independently of the eddy motions; rather, the two are intimately related.

The theoretical description of this mean cell-eddy "compensation" has recently received new light with the development of a "Lagrangian-mean" dynamical formulation (Andrews and McIntyre, 1976, 1978a,b). Although much of the application of these Lagrangian ideas to the atmosphere is yet to be done, an important paper already published is that of Kida (1977) who has discussed a GCM and its related Lagrangian-mean circulation. By calculating a large number of parcel trajectories in his model, Kida has shown qualitatively the sense of mass flow in the troposphere and lower stratosphere. In doing so, Kida has emphasized the distinction between this Lagrangian-mean meridional circulation and the corresponding, but quite different, Eulerian-mean circulation. As several authors have pointed out, the physical mechanism which causes this Lagrangian-mean meridional circulation to differ from its Eulerian counterpart is the Stokes drift (Stokes, 1847; Longuet-Higgins, 1967; Moore, 1970; Andrews and McIntyre, 1978a; Wallace, 1978).

That the Stokes drift causes these two circulations to differ from one another is quite significant. There do exist, in fact, certain situations in which these circulations are not only different, but also of opposite sign. One such situation appears to have been observed several years ago by Riehl and Fultz (1957) who discussed a "jet stream axis" coordinate system in connection with rotating tank experiments. Those authors found that a mean circulation derived by averaging along streamlines was directed opposite to the more conventional zonally averaged cell. A similar result was found by Mahlman (1969) who discussed the analogous circulations present during a stratospheric sudden warming.

It will be argued here that such a situation also prevails throughout the winter season in the extratropical stratosphere and mesosphere. As a result, the existence of a Brewer–Dobson cell will be shown to be entirely consistent with the presence of a two-cell Eulerian-mean meridional circulation.

The present study, then, is an attempt to investigate analytically the general sense of mass transport in the stratosphere and mesosphere. The present results effectively complement the work of Kida (1977) who discussed motions only below 100 mb. Of course, we must face the question as to how a net transport by the mean flow and eddies is accomplished. The present paper shows that this net transport is achieved entirely by a Lagrangian-mean flow, and that this Lagrangian-mean flow is, under suitable approximations, obtainable from a knowledge of Eulerian mean diabatic heating.

The steps leading to the derivation of this Lagrangian-mean circulation are really quite simple; however, the direct derivation based on concepts developed in the work of Andrews and McIntyre (1978a) is relegated to the Appendix. The main body of this paper takes up a more gradual development of this subject, with careful consideration of the concepts involved, as well as a close view of the limitations inherent in the approximations made throughout the paper. In Section 2, a simple transformation of variables is introduced into the thermodynamic and continuity equations. It is shown that, under the approximations made, the transformed variables may be determined by diabatic heating alone. It is next shown, in Section 3, that under

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1 The Lagrangian-mean employed in this paper differs, in general, from this "jet-stream" average; however, the two averages become approximately equivalent if the streamlines are steady and stationary.

2 On the other hand, Krishnamurti (1961) found these two circulations to be of the same sign in the tropical atmosphere, presumably because of the relative weakness of the eddies there.
additional approximations, the transformed variables represent the Lagrangian-mean meridional circulation. Of course, it is necessary to discuss precisely what is meant by a Lagrangian mean, and this question is also given careful consideration in Section 3. In Section 4 the resulting circulation, based on Murgatroyd and Goody's (1958) heating rates, is presented. The precise interpretation of this circulation is reserved for Section 5, where a net annual circulation is qualitatively considered.

2. Transformation of variables

Following Murgatroyd and Singleton (1961) this paper employs the thermodynamic equation and the equation of continuity, which in spherical, log-pressure coordinates are of the form (Holton, 1975)

\[ \frac{\partial T}{\partial t} + \frac{\partial T}{\partial \phi} + \left( \Gamma_d - \Gamma \right) \tilde{w} = \tilde{Q} - \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left( \frac{v^* T^* \cos \phi}{\rho_0 \Gamma_d} \right), \]  (2.1)

\[ \frac{1}{\rho_0 \cos \phi} \frac{\partial}{\partial \phi} \left( \overline{v \cos \phi} \right) + \frac{1}{\rho_0 \Gamma_d} = 0. \]  (2.2)

Here an overbar refers to a zonal average and a prime, the deviation therefrom. \( T \) is the temperature; \( v \) and \( w \) are meridional and vertical velocities, respectively; \( \Gamma_d - \Gamma \) is a measure of static stability; \( \rho_0 \) is a basic-state density; \( H = i \) km; \( \tilde{Q} \) is the Eulerian mean diabatic heating; and \( a, \phi \) and \( z \) are the radius of the earth, latitude and height, respectively.

Eq. (2.1) may be simplified with the following approximations:

1) Attention is focused on solstice conditions, at which times the term \( \partial T/\partial t \) may be considered negligibly small.

2) Vincent (1968) found the advection term \( \left( \tilde{w}/a \right) \left( \partial T/\partial \phi \right) \) to be small in the lower stratosphere. Here we assume it to be small also in the upper stratosphere and mesosphere, an approximation which will be justified a posteriori in Section 4.

3) Because of the nearly quasi-geostrophic character of wave motions in the upper atmosphere outside of the tropics, we expect that vertical eddy fluxes are \( O(\text{Ro}) \). This is supported by Vincent's study, who found the vertical eddy heat flux to be negligibly small.

After these approximations, Eq. (2.1) becomes

\[ \left( \Gamma_d - \Gamma \right) \tilde{w} = \tilde{Q} - \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left( \frac{v^* T^* \cos \phi}{\rho_0 \Gamma_d} \right) \]  (2.3)

3. Identification and interpretation of Lagrangian-mean velocities

Andrews and McIntyre (1978a) have discussed a "generalized Lagrangian-mean" formulation of the primitive equations. In their paper, the word "mean" may refer to a host of averaging operators, among which are the ensemble average, time average and zonal average. Here we restrict all attention to the zonal average. In that case, a suitable interpretation of the Lagrangian mean, due to Andrews and McIntyre (1978a), is as follows. Consider a string of fluid parcels initially lying along the \( z \) axis (the direction of averaging). Suppose that imaginary "elastic bands" are used to connect each parcel to an imaginary "massless rod" initially coincident with the parcels. Fig. 1 shows that as the flow evolves, parcels are displaced from their initial positions, and hence exert a force on the rod, the end result at which we are aiming does not require \( \left( \Gamma_d - \Gamma \right) \) to be constant as seemingly required here (see Appendix).
possibly causing it to move. To motivate the use of zonal averaging, the rod is constrained (by a magnetic field, say) to remain parallel to the $x$ axis. The Lagrangian mean for each such rod is defined as the average over all the parcels (formally, an infinite number of parcels) associated with that particular rod. For example, the Lagrangian-mean velocity is just the velocity of the rod itself.

Andrews and McIntyre (1978a) have demonstrated that the use of this concept of Lagrangian-mean renders simplification to the mean form of the primitive equations. Matsuno and Nakamura (1978) have discussed some physical mechanisms underlying this theory by considering the stratospheric sudden warming phenomenon from a Lagrangian viewpoint. The present paper is not concerned specifically with wave-mean flow interaction theory, but rather with transport. Wallace (1978) has discussed the importance of the Stokes drift for transport; the present paper uses the ideas developed in that paper, but we here aim at a description of the total Lagrangian-mean flow (following Kida, 1977), of which the Stokes drift forms only a part.

We consider the “linearized” version of Lagrangian-mean theory as described by Andrews and McIntyre (1976). [The finite-amplitude theory, using ideas developed by Andrews and McIntyre (1978a), is presented in the Appendix.] For example, the linearized perturbation thermodynamic equation is [for simplicity, the following discussion assumes Cartesian geometry; spherical geometry requires only slight modifications (Andrews and McIntyre, 1978b)].

$$
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \left[ T' + \frac{\partial T}{\partial y} \right] \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y} \left[ \psi \left( \Gamma_d - \Gamma \right) \right] = 0.
$$

A small-amplitude meridional displacement field $\eta'$ is introduced as

$$
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \eta' = \psi'.
$$

By multiplying Eq. (3.1) by $\eta'$ and taking the zonal average, we find that

$$
\frac{\bar{\eta} T' - \bar{\psi} T'}{\eta'} - \frac{1}{2} \frac{\partial}{\partial y} \frac{\partial \psi}{\partial \psi} = \frac{\partial}{\partial y} \left[ \eta \left( \Gamma_d - \Gamma \right) \right] = \eta' Q'.
$$

As discussed in the Introduction, the non-interaction theorem is valid for waves which are steady and conservative. For these waves, we differentiate Eq. (3.3) with respect to $y$ to obtain

$$
\frac{\partial}{\partial y} \left( \eta' \psi' \right) = \frac{\partial}{\partial y} \left( \frac{\psi' T'}{\Gamma_d - \Gamma} \right).
$$

Following Andrews and McIntyre (1978a), the term on the left-hand side is to be recognized as the leading approximation to the vertical Stokes drift $\bar{\psi}$. This term arises because of differences inherent in the Eulerian and Lagrangian methods of averaging. For small-amplitude waves the spatial and temporal gradients of wave amplitude and phase contribute to this term in a manner originally discussed by Stokes (1847) for surface gravity waves, and more recently by Longuet-Higgins (1967), Moore (1970) and Wallace (1978). In the present case, it is the meridional gradient of wave amplitude which is of importance. With this identification, Eq. (3.4) becomes

$$
\bar{\psi} = \frac{\partial}{\partial y} \left( \frac{\psi' T'}{\Gamma_d - \Gamma} \right).
$$

Because the Lagrangian mean of any quantity is the sum of the Eulerian mean of the quantity and its corresponding Stokes correction, we then have for example,

$$
\bar{\psi}^L = \bar{\psi} + \bar{\psi}^S.
$$

Because $\bar{\psi}^L$ describes the motion of the “massless rod” of Fig. 1, it provides us with a suitable definition of mass transport. Although individual fluid parcels, in the presence of eddies, do not move with this velocity, their centers of mass (the rods) will. The identification of this Lagrangian-mean velocity with mass transport is particularly well-suited to the linearized, small-amplitude waves discussed in this paper, since such wave motions merely imply small oscillations of individual parcels about their respective “rods.”

Furthermore, we now have the rationale for the transformation of variables made in Section 2. Eqs. (3.5), (3.6) and (2.4) may be combined to show that, for steady and conservative waves,

$$
\bar{\psi}^S = \psi'.
$$

This relation, in conjunction with Eq. (2.6), gives the desired result

$$
\bar{\psi}^L = \left( \frac{\partial}{\partial y} \right) \left( \Gamma_d - \Gamma \right).
$$

Eq. (3.8) states that, under all the approximations made in Sections 2 and 3, the Lagrangian-mean vertical velocity, and hence vertical mass transport, are determined to leading order by the Eulerian-mean diabatic heating and static stability alone.

One may proceed in a similar manner to derive the Lagrangian-mean meridional velocity $\bar{\psi}^L$. For the present, however, it is sufficient to note that, under the approximations made in Sections 2 and 3, the Lagrangian-mean meridional circulation satisfies the continuity equation

$$
\frac{1}{\rho_0} \frac{\partial}{\partial \phi} \left( \rho \psi \cos \phi \right) + \frac{1}{\rho_0} \frac{\partial}{\partial z} \left( \rho \psi \bar{\psi}^L \right) = 0.
$$

Eqs. (3.8) and (3.9), then, constitute an approximate set which may be used to derive the Lagrangian-mean meridional circulation. The following sections discuss
a circulation derived with these equations. Naturally, the validity of this circulation depends on the accuracy of the "steady and conservative waves" assumption. As discussed in the Introduction, in the Lagrangian-mean equations, inhomogeneous terms which force a Lagrangian-mean meridional circulation strictly involve wave transience and dissipation (as opposed to eddy fluxes). Thus, if wave transience and wave dissipation are present, the actual Lagrangian-mean circulation may differ from such a circulation inferred from zonal-mean diabatic heating alone.

The best argument in support of the neglect of these terms appears a posteriori to lie in the fact that a Lagrangian-mean meridional circulation forced by zonal-mean diabatic heating (as discussed below) agrees qualitatively with the Brewer–Dobson cell, which is representative of many features of tracer transport. Of these two terms, wave transience appears to be more important (Holton and Dunkerton, 1978).\(^6\) Wave dissipation, on the other hand, is small relative to \(Q\); the leading contribution to this term involves \(\nabla T'\) [see Eq. (3.3)] which is small since (i) \(|T'| < |Q|\), reflecting the fact that \(|T' < T - \bar{T}_e|\), i.e., if the diabatic heating is represented as a "return to equilibrium" heating, this heating is dominated by the zonal-mean contribution; and (ii) for stratospheric planetary waves, \(v'\) and \(T'\) are well-correlated, which suggests that \(\nabla'\) and \(Q\) are in quadrature. As a result, the Stokes correction to the diabatic heating \(\bar{Q}^d\) is relatively small compared to \(Q\).

4. Calculation of radiatively driven Lagrangian-mean velocities

To digress slightly, it is worth reiterating that Lagrangian-mean circulations, one of which this paper seeks to derive, are driven fundamentally by forcing mechanisms which are themselves Lagrangian-mean mechanisms. In the present case, the above remarks establish that, if the stratosphere-mesosphere Eulerian-mean meridional circulation is driven by a diabatic heating distribution similar to that used by Murgatroyd and Singleton, the Stokes correction to this distribution may be neglected. In this and the following sections the Lagrangian-mean circulation driven by this heating is examined, along with some of its implications for tracer motions. Undoubtedly, however, future studies will be required to deal with the quantitative aspects of this problem. In particular, more accurate diabatic heating rates will be required.\(^6\) Such rates would best be evaluated using a fully three-dimensional radiative transfer model in the presence of a wave, since this method would allow for the implicit determination of \(Q\).

Radiative heating rates for the solstices were taken from Fig. 1 of Murgatroyd and Singleton (1961). Static stability was estimated indirectly from the \textit{U. S. Standard Atmosphere Supplements} (1966).

Fig. 2 shows the Lagrangian-mean vertical velocity \(\bar{v}^L\) calculated from Eq. (3.8). Eq. (3.9) was then integrated to find \(\bar{v}^L\) assuming \(\bar{v}^N = 0\) at \(\phi = \pm \pi/2\).\(^7\) Fig. 3 shows the \(\bar{v}^L\) calculated in this manner.

The vertical velocities shown in Fig. 2 are very similar in structure to the diabatic heating rates, although comparison of Fig. 2 to Fig. 1 of Murgatroyd and Singleton (1961) shows that the reduced static stability of the summer mesosphere has doubled \(\bar{v}^L\) over what would be expected from \(Q\) alone. Consistent with certain features of tracer transport, there is weak rising motion over the tropics, accompanied by subsidence into the extratropical troposphere. Associated with the \(O_3\) heating and \(CO_2\) cooling maxima near the stratopause are strong rising and sinking motions, respectively.

The meridional velocities required to balance \(\bar{v}^L\) show an outflow from the tropical lower stratosphere into the winter and summer polar stratosphere. In the mesosphere there is a strong flow from the summer to winter hemisphere which is prominent at higher levels because of the reduced atmospheric density there.

Using the calculated velocities, the advection term \(\bar{v}^L (\partial \bar{T}/\partial y)\) may be estimated. Largest relative values occur in the mesosphere; however, the magnitude of this advection would be about 1 K day\(^{-1}\), which is generally insignificant compared to \(Q\). Consequently, by ignoring this term we obtain a qualitatively correct estimate of \(\bar{v}^L\) and \(\bar{w}^L\) which compares well to the

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\(^6\) This point has been brought to the author’s attention by J. D. Mahlman and R. S. Lindzen.

\(^7\) A very small adjustment of \(\bar{v}^L\) (about 1%) was required in order to make \(\bar{v}^L\) satisfy these boundary conditions.
circulation of Murgatroyd and Singleton (1961) who included this term.

5. Interpretation of results

Streamlines associated with the calculated Lagrangian-mean velocities are shown in Fig. 4. This figure is very similar to Fig. 1b of Hesstvedt (1964) who constructed such a diagram based on Murgatroyd and Singleton's results.

However, Fig. 4 must be interpreted with caution, as it represents a radiatively driven Lagrangian-mean flow for only three months of the year. Because much of the figure is antisymmetric about the equator, the net annual mass transport may be substantially different.

The present paper considers a qualitative "two-cycle" model to determine this net transport. This model assumes quiet equinoctial conditions for three months with an equal and opposite circulation to that of Fig. 4 for three more months, followed by three months of equinoctial conditions and so on.

Selected trajectories of "massless rods" are visualized in Fig. 5. From this figure we observe the following:

1) Air parcels initially moving poleward (A,B) when leaving the tropical troposphere arrive in the polar lower stratosphere within one to two years.

2) Air parcels injected directly upward into the tropical stratosphere (C) tend to drift poleward, arriving in polar latitudes after two to three years.

3) This poleward drift becomes slower at higher levels (D) while mesospheric and polar upper stratospheric parcels merely oscillate between hemispheres (E).

The direction of this net mass circulation in the lower stratosphere is in qualitative agreement with the observed distributions of ozone\(^8\) and water vapor (Dutsch, 1971; Stanford, 1974), and is not inconsistent with the observed "poleward and downward" spreading of radioactive and volcanic debris (e.g., Feely and Spar, 1960; Reed and German, 1965; Dyer and Hicks, 1968). The magnitude of this flow near 100 mb is also suggestive of one-year "residence times" for lower stratospheric air implied by these same radioactive debris observations (e.g., Nydal, 1968).\(^9\)

One particularly pertinent observation supporting the mesospheric circulation discussed here has been examined by Kalkstein (1962). In the autumn of 1958

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\(^8\)The ozone distribution, indeed, constitutes perhaps the strongest evidence for the Lagrangian circulation discussed here. A curious paper is that of Prabhakara (1965) who, albeit unknowingly, demonstrated this point by reproducing the observed O\(_3\) distribution in a transport model which employed a combination of Murgatroyd and Singleton's (1961) mean meridional circulation and "turbulent diffusion."

\(^9\)The question arises as to how an equinoctial circulation would affect these results. Assuming an equinoctial cell like that of Murgatroyd and Singleton (1961), one might expect the lower stratospheric residence times calculated with our two-cycle model to be too long. However, it is felt that this effect would be partially, if not entirely, offset if the present calculations were repeated using the more accurate heating rates of Dopplick (1972), which as a rule, are about half of the Murgatroyd and Goody (1958) rates in the lower stratosphere.
a large quantity of rhodium 102 was released into the tropical upper stratosphere and mesosphere by a nuclear explosion over Johnston Island (16°N, 170°W). Subsequent to the explosion the rhodium debris concentration was observed to increase substantially in the Southern Hemisphere in June of 1959, and then to increase even more dramatically in the Northern Hemisphere approximately six months later. Three years after the explosion, rhodium concentrations above 20 km were still almost an order of magnitude smaller over tropical latitudes (20°N–20°S) than elsewhere. This evidence gives qualitative support to the Lagrangian-mean mesospheric circulation discussed in this paper, which is essentially characterized by yearly hemispheric exchange at high levels with little net transport into the lower tropical stratosphere from above.

6. Conclusion

The distinction between Eulerian and Lagrangian circulations has been stressed in the present paper. This distinction has only recently been made clear for meteorological applications (Kida, 1977; Wallace, 1978) even though the basic physical mechanisms underlying the two circulations were first discussed by Stokes (1847) over 130 years ago.

A schematic mean meridional circulation of the upper atmosphere at the solstices is shown in Fig. 6. It can be seen that wintertime eddies serve to make the Eulerian circulation quite different from its Lagrangian counterpart. The downward Eulerian motion in mid-latitudes is consistent with results of Vincent (1968) and Hartmann (1976) for Northern and Southern Hemisphere winters, respectively. What needs to be stressed, however, is that this Ferrel cell poleward of 40° has no net effect on mass transport. This is because a Ferrel cell, in the absence of diabatic heating, is a wave-induced cell; it exists only in order to cancel the Stokes drift associated with the eddies. Such cancelation is necessary because the Stokes drift by itself would act to bring the zonal-mean winds out of thermal wind balance (McIntyre, 1977). Thus we have a Lagrangian explanation of the near-cancelation between eddy and mean transports (see Introduction). The “leftover” transport (for waves which are steady and conservative) is accomplished by diabatic heating alone.

In closing, it is useful to consider how the analysis of the present paper may be improved. One obvious improvement would be to include, either explicitly or in a parameterized fashion, the effects of wave transience and dissipation. A second and more important improvement would involve the consideration of a Lagrangian-mean momentum budget. One way to incorporate this momentum budget into a Lagrangian-mean system of equations is to assume that the Eulerian-mean flow is in thermal wind balance. Eliassen (1950) has considered this problem in an Eulerian system of equations, the result of which is that one obtains an elliptic equation for the Eulerian-mean meridional circulation of the form

\[
\vec{v}_{yy} + \vec{v}_{zz} = (\text{terms involving Eulerian-mean forcing and dissipation}) + (\text{eddy flux terms}).
\]

(6.1)

Not surprisingly, an analogous equation may be derived in the Lagrangian-mean system which is of the form

\[
\vec{v}_{yy} + \vec{v}_{zz} = (\text{terms involving Lagrangian-mean forcing and dissipation}) + (\text{wave transience terms}).
\]

(6.2)

An equation of this form has, in fact, been employed by Matsuno and Makamura (1978) who investigated the Lagrangian mean flow during a sudden warming. Generalizations of their work to include diabatic heating and seasonal effects, then, would be of considerable interest to the topic discussed in the present paper.

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APPENDIX

Transport by Finite-Amplitude Waves

Andrews and McIntyre (1978a) have discussed the "generalized Lagrangian mean" and its implications for wave-mean flow interaction theory. The present dis-
cussion focuses on the relevance of this Lagrangian mean for the transport theory considered in this paper. In the light of the following discussion, the approximations introduced throughout the main body of the paper will be made clearer. Consequently, the limitations of the present work, which were pointed out in Section 6, will also be made more obvious.

Following the derivation in Section 2, we assume an exact thermodynamic equation of the form

$$\frac{\partial \theta}{\partial t} = Q,$$

where $\theta$ represents potential temperature. By taking the Lagrangian mean of Eq. (A1), we obtain

$$\bar{\theta} = \bar{Q},$$

where

$$\bar{\theta} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla$$

is the time rate of change following the “massless rod” of Fig. 1. The approximations discussed in the main body of the paper are conveniently summarized as follows:

1) Neglect of $\partial \bar{\theta}/\partial t$; i.e., time-averaged solstice conditions.

2) Neglect of $\bar{Q}$ ($\partial \bar{\theta}/\partial y$); an approximation justified a posteriori in Section 4.

3) Neglect of $\bar{Q}$; the leading eddy contribution to this term was shown to be relatively small in Section 3.

4) Neglect of $\bar{v}$; wave transience and “turbulent diffusion” contribute to this term.

The final result, employed in Section 4, is

$$\frac{\partial \bar{\theta}}{\partial z} = \bar{Q},$$

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