The structure of zonal jets in shallow water turbulence on the sphere

R. K. Scott

1 Introduction

The large-scale motions of planetary atmospheres and oceans, constrained by strong stable stratification and rapid rotation, are characterized in part by quasi-two dimensional turbulent motion and in part by coherent structures, such as vortices, zonal jets, and low-frequency waves. These motions are intimately linked and understanding their complex and multiscale interactions presents a complex and formidable challenge. Broadly speaking, turbulent motions can be thought of as occurring primarily at small scales and zonal jets and wave motions at large scales. However, turbulence is organized into latitudinal bands by these zonal jets, while the jets themselves are maintained against dissipation by turbulent and wave motions, via eddy momentum fluxes (e.g., McIntyre, 1982; Baldwin *et al.*, 2007, and references therein). Examples of zonal jets include the winter polar night jet in the Earth's stratosphere and the sub-tropical jet stream (both driven diabatically but intensified by wave/turbulent processes), and, perhaps most famously, the jets responsible for the characteristic banded structure observed on the giant planets. These zonal jets are instrumental in determining the transport of heat, chemical tracers, kinetic energy and angular momentum. Transport is rapid along the jet axis, but is strongly inhibited across the jet axis due to enhanced "Rossby wave elasticity", the dynamic resiliance of jets to latitudinal displacements, and strong latitudinal shear on the jet flanks. Transport of water vapor within the troposphere, for example, a major influence on climate, is sensitive to the latitudinal position of the atmospheric jet stream. In the stratosphere, details of wave breaking in the vicinity of the polar night jet determine the meridional transport of chemical species involved in ozone chemistry and ultimately determines the rate of ozone recovery (World Meteorological Organization, 2007). On the giant planets, jets separate regions of different trace chemical composition, giving rise to the coloured bands.

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The emergence of zonal jets in geostrophic turbulence can be associated with the action of Rossby waves in arresting the upscale cascade of energy in a two-dimensional turbulent flow (Rhines, 1975; Williams, 1978). As energy accumulates at larger scales and lower frequencies, a point is reached where wave motions become significant and their capacity for long-range momentum transport leads to zonal accelerations and the emergence of well-defined zonal jets. In the simple geometry of the β-plane, where planetary rotation varies linearly about a typical mid-latitude value, zonal jets possess a well-defined latitudinal structure (e.g., Maltrud & Vallis, 1991; Huang & Robinson, 1998; Danilov & Gurarie, 2004) with a direct relationship between jet strength and spacing that is now reasonably well understood. At scales for which advection of relative vorticity ζ dominates advection of planetary vorticity, the gradient of planetary rotation β can be neglected and a timescale for the motion at scale $L \sim k^{-1}$ can be taken as $\tau \sim (kU)^{-1}$, where k is a wavenumber, or inverse length scale and U is a typical velocity (see, e.g., Vallis, 2006, for an overview). In such a flow, classical phenomenology predicts that energy cascades to ever larger scales, coherent vortices merge and grow. However, the advection of planetary vorticity also supports the propagation of Rossby waves with frequency ω satisfing the dispersion relation $\omega = k\beta/(k^2 + l^2)$ (although this must be modified when significant zonal motions are present), where now k and l are zonal and latitudinal wavenumbers, respectively. These waves will be excited if there is an overlap between the frequency of the turbulent motion with the frequency of the waves, i.e. when $\tau \sim \omega^{-1}$, and this occurs at the Rhines scale, $L_{\mathrm{Rh}} \sim \sqrt{U/\beta}$. Low frequency wave motions and accompanying radiation stresses make possible long range transport of zonal momentum and thereby give rise to coherent zonal jets, providing an effective halt to the inverse energy cascade at a Rhines scale L_{Rh} (Rhines, 1975; Williams, 1978).

These predictions have been generally well supported by many numerical, experimental and observational studies. In particular, numerical calculations have verified the direct relation between jet spacing and either the initial energy in the freely decaying case (e.g. Yoden & Yamada, 1993; Cho & Polvani, 1996a,b; Iacono *et al.*, 1999), or the forcing amplitude in the continuously forced case (e.g. Vallis & Maltrud, 1993; Nozawa & Yoden, 1997; Huang & Robinson, 1998). Typically zonal jets are found that are very steady in time. Other studies have demonstrated a similarity between power spectra of simulations and observations from the giant planets (Galperin *et al.*, 2001; Huang *et al.*, 2001; Sukoriansky *et al.*, 2002) or considered the effect of dissipation on jet spacing (Smith, 2004; Sukoriansky *et al.*, 2007).

The remainder of this paper reviews and updates some recent results of shallow water turbulence on the surface of a rotating sphere, examining, separately, the effects of compressibility or free surface deformations together with the associated means of energy dissipation (Scott & Polvani, 2007, 2008) and the consequences of global angular momentum and barotropic stability constraints (Dunkerton & Scott, 2008), of which the latter provides a simple relation between jet spacing and jet strength that is independent of the phenomenology reviewed above. In particular, and as discussed further below, we venture the suggestion that the deformation radius appropriate to jets on the giant planets may be larger than usually considered, based on an examination of the degree of zonal alignment of high latitude jets in numerical simulations. We also summarize the recent result of Scott & Polvani (2008) that a representation of large-scale energy dissipation through radiative process, or thermal damping, leads robustly to superrotating equatorial jets, even in the case of small deformation radius.

2 Jet undulations

Including the effects of compressiblility introduces another length scale, the deformation radius, L_D, representing the scale on which gravitational and rotational forces are comparable. In the Earth's troposphere $L_D \approx a$, the planetary radius, in the oceans $L_D \approx 0.1a$, while in the atmospheres of Jupiter and Saturn $L_D \approx 0.025a$ (e.g. Cho *et al.*, 2001; Ingersoll *et al.*, 2004). Purely 2D flow is recovered in the limit $L_D \to \infty$. Accounting for finite L_D alters the Rossby wave dispersion relation and suggests a modified Rhines scale $L_{\rm Rh} \sim \sqrt{U/\beta(1-\alpha)}$ where $\alpha = U/\beta L_D^2$. This means that when $L_D^2 < U/\beta$ (i.e. $\alpha > 1$), there will be no overlap between the turbulent and wave motions and that the flow will remain isotropic (Okuno & Masuda, 2003; Smith, 2004; Theiss, 2004). On the sphere L_D varies with latitude, increasing toward the equator; therefore, for certain parameters a dual regime may exist comprising isotropic 2D turbulent motion at high latitudes and coherent jets at low latitudes.

Fig. 1 shows snapshots of relative vorticity from simulations with random isotropic vorticity forcing at small scales, similar to those described in Scott & Polvani (2007) but repeated here at higher resolution, and illustrates the influence of the latitudinal variation of L_D on the emerging structure of goestrophic turbulence. Sharp jumps in values correspond to eastward jets. In (a), where $L_D/a = 1$ at the pole, jets are visible at all latitudes across the sphere and are strongly zonal. By contrast, in (b), where $L_D/a = 0.025$ at the pole, jets only appear at low and mid-latitudes. At higher latitudes the jets become increasingly undular until, in polar regions there are no jets at all, and instead the flow is dominated by circular vortices. This behaviour is consistent with the scaling arguments just presented (see Scott & Polvani, 2007, for further discussion).

At values of deformation radius typical of the giant planets planets, numerical experiments consistently produce high latitude jets that are significantly more undular than their planetary counterparts. Only at values of deformation radius double or more the planetary values do we begin to see stongly zonal jets at mid and high latitudes. Estimates of deformation radius for the giant planets are, however, typically inferred from observations or numerical modelling of the structure of coherent vortices such as the Great Red Spot (e.g. Cho *et al.*, 2001; Ingersoll *et al.*, 2004). The aforementioned numerical experiments suggest, therefore, that jets may have a deeper structure, and a correspondingly larger effective deformation radius, than smaller scale vortices. Such a structure may arise naturally through the process of barotropization and is observed to occur in many flows in the terrestrial atmosphere

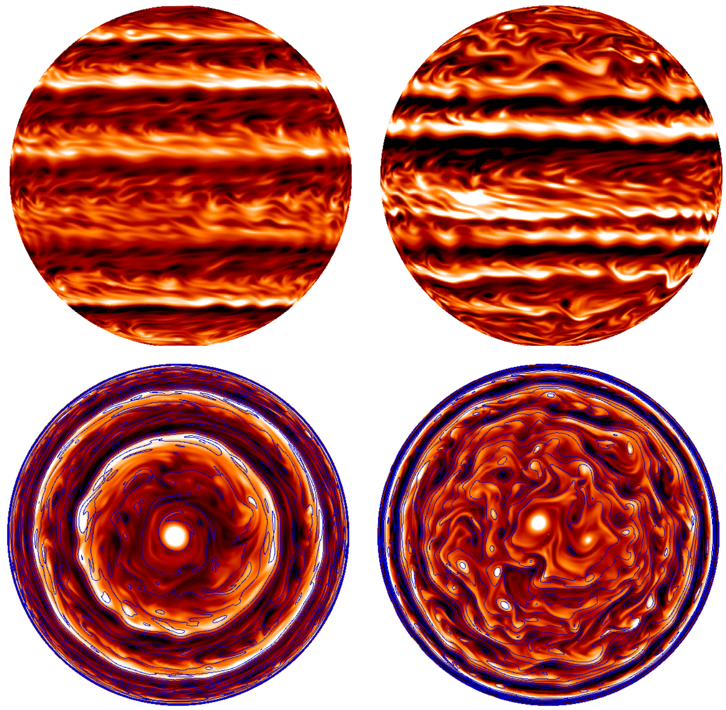

Fig. 1 Illustration of the latitudinal confinement of zonal jets: $L_D/a = 1$ (left), $L_D = 0.025$ (right); shaded quantity is the relative vorticity, the contours in the lower panels show the potential vorticity. Top: equatorial view; bottom: polar view.

and oceans. In the above experiments, high latitudes cease to be strongly zonal when L_D/a is below around 0.1.

Although the polar views of relative vorticity field shown in Fig. 1 appear increasingly isotropic at small L_D/a, an examination of the potential vorticity field (blue contours) indicates the flow is not isotropic. Contours of potential vorticity are highly undular but nevertheless circumscribe the pole. Interestingly, the strong coherent cyclonic vortices seen near the pole are a persistent feature of these and many other numerical experiments and bear some resemblence to the polar cyclones observed on Jupiter and Saturn.

3 The potential vorticity staircase

Several recent studies (Baldwin *et al.*, 2007; Dritschel & McIntyre, 2008; Dunkerton & Scott, 2008; McIntyre, 2008) have emphasized the dual role of turbulent mixing and long-range interactions, the latter arising from the property of potential vorticity inversion and including long-range momentum transport by waves. In fact, the idea of inhomogeneous mixing of potential vorticity in the context of Rossby wave–mean flow interaction appeared over 25 years ago in the McIntyre (1982) review of stratospheric sudden warmings and the related discussion by McIntyre & Palmer (1983) of a midlatitude "surf zone" bounded on both sides by sharp gradients of potential vorticity and tracer. Material conservation and invertibility of potential vorticity give rise to a "Rossby-wave elasticity" or resiliance to latitudinal displacements of regions of strong potential vorticity gradients. Mixing of potential vorticity is therefore highly inhomogeneous: stronger gradients have greater elasticity and so resist mixing by eddy or wave motions; where gradients are weaker, mixing is stronger and contours of constant potential vorticity can be more easily deformed irreversibly. Mixing here means latitudinal mixing across the planetary vorticity gradient and thus acts to further weaken local potential vorticity gradients in mixing regions, and strengthen them in between. Zonal jets are a direct consequence of the resulting potential vorticity staircase structure through the relation

$$\bar{q} = -\frac{1}{a^2}\frac{d\bar{m}}{d\mu} \tag{1}$$

where $\mu = \sin\phi$, ϕ is latitude, $\bar{m} = a\cos\phi(\Omega a\cos\phi + \bar{u})$ is the angular momentum, $\bar{u}$ is the zonal velocity, Ω is the planetary rotation, and the overbar denotes a zonal mean. The potential vorticity staircase solution corresponds to latitudinal bands of constant $\bar{q}$ separated by jumps, as shown in Fig. 2. According to (1), uniform $\bar{q}$ implies that $\bar{m}$ is linear in μ. The associated $\bar{u}$ is then obtained from the departure of $\bar{m}$ from the parabolic profile of the resting atmosphere. The construction shown here follows that of Dunkerton & Scott (2008). Note that the staircase construction gives an immediate explanation, which is hidden in the phenomenological arguments reviewed in §1, of the asymmetry between sharp, narrow prograde jets and broad, weak retrograde flow generically observed in geostrophic turbulence.

If further we assume a rearrangement of potential vorticity from a resting atmosphere that conserves total angular momentum and assume that the resulting flow be barotropically stable, then the staircase construction also provides an explicit constraint on the jet spacing for a given jet strength: geometrically, it is easy to see that we may either have a few strong jets or many weak ones, as illustrated by the two rows in Fig. 2. We may term the scale arising from this constraint a "geometric Rhines scale" (Dunkerton & Scott, 2008), since it arises independently of any details of the turbulent phenomenology or potential vorticity mixing. It complements the notion of dynamical Rhines scale discussed in §1 (Rhines, 1975) or similar estimates based on the spectral flux of energy (Maltrud and Vallis, 1991), but is conceptually simpler, requiring only that potential vorticity be homogenized within mixing zones located between prograde jets, in a barotropically stable configuration. Further analysis shows that jets are spaced slightly further apart than predicted by the arguments of §1. Since the staircase solution assumes perfect homogenization of potential vorticity between jets it should also be considered as a limiting situation.

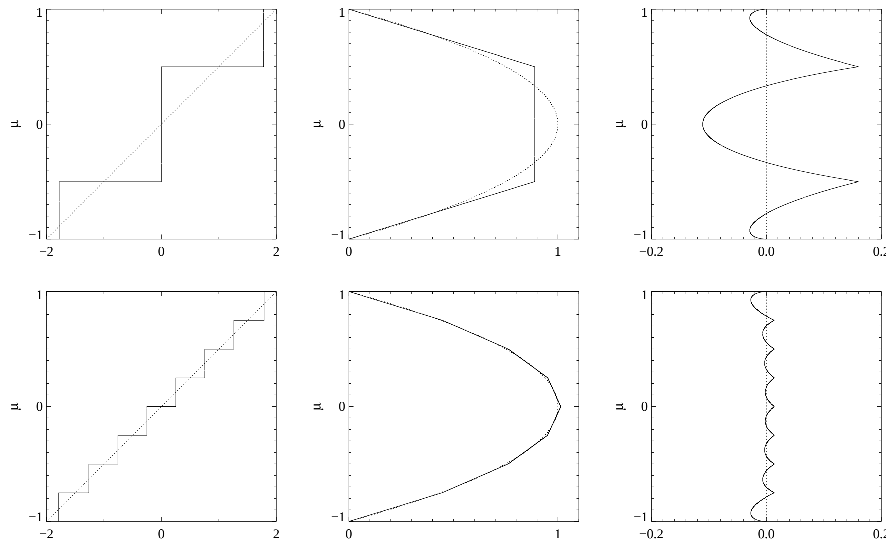

Fig. 2 Illustration of the potential vorticity staircase and the geometric Rhines constraint; top, few large steps; bottom, many small ones; $\bar{q}$ (left), $\bar{m}$ (centre) $\bar{u}$ (right), against $\mu = \sin\phi$, where ϕ is latitude. Dotted lines correspond to an atmosphere at rest.

The β-plane case with finite L_D/a was discussed in Dritschel & McIntyre (2008). There, exact solutions were found for the staircase solution, which led to the estimate $L_{\text{jet}} \sim \max(L_{\text{Rh}}, L_{\text{Rh}}^2/L_D)$ for the jet spacing. In other words the jet spacing scales as L_{Rh} at large L_D, but is modified for $L_D \ll L_{\text{Rh}}$. Note, however, that in the latter case, zonal jets are typically undular, similar to the high latitudes of Fig. 1, and their identification becomes increasingly difficult, since, for example, zonal averaging will smooth out the along jet velocity.

The scaling for the barotropic case on the sphere was confirmed to hold over a wide range of energy injection rates (Dunkerton & Scott, 2008). A further example is shown in Fig. 3, for a case of finite deformation radius with small-scale forcing and large-scale energy dissipation by thermal relaxation. In this case, the potential vorticity profile is close to the limiting case of the perfect staircase, and the corresponding jet asymmetries are apparent. The full potential vorticity field is also shown and illustrates the turbulent mixing between the locations of strong gradients.

4 Equatorial superrotation

With physically appropriate parameters, Cho & Polvani (1996b) showed that the scale of zonal jets in freely decaying shallow water turbulence was consistent with the observed zonal mean winds of the four giant planets. However that study, and

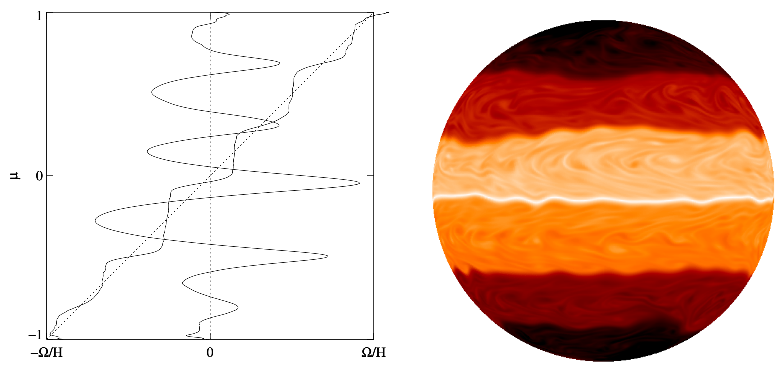

Fig. 3 Forced numerical staircase for a case with $L_D = 0.05$ and radiative relaxation: left, $\bar{q}$ and $\bar{u}$; right, the potential vorticity.

many others since have found that equatorial jets tend to be subrotating, or retrograde, when L_D/a is small, in contrast to the equatorial jets observed on Jupiter and Saturn. Although this result has sometimes been used in arguments for the role of deep convection as the driving mechanism for the observed jets, it is important to remember that subrotating jets are only predominant at small L_D/a: when L_D/a is large subrotating and superrotating equatorial jets appear to emerge with approximately equal probability when forced randomly, as was demonstrated in an ensemble of calculations performed by Dunkerton & Scott (2008).

The staircase construction discussed above is again useful in understanding the nature of the equatorial jets. Whether an equatorial jet is superrotating or subrotating is determined simply by whether the equator coincides with a potential vorticity jump or mixing zone. The emergence of equatorial superrotation in a non-axisymmetric, turbulent flow can be understood in terms of the Taylor identity (McIntyre, 2008), relating eddy potential vorticity fluxes and eddy momentum flux convergence, which, in the simplest case of barotropic motion, takes the form

$$\overline{v'q'} = -\frac{1}{a}\frac{d}{d\mu}\left(\overline{u'v'}\sqrt{1-\mu^2}\right). \tag{2}$$

Advective mixing of potential vorticity is always associated with a downgradient potential vorticity flux or divergence of eddy momentum flux, and a consequent reduction of angular momentum and deceleration of the zonal velocity. To establish a potential vorticity jump at the equator, a nonadvective upgradient flux is necessary, giving rise to a local acceleration of the zonal velocity: positive jets must necessarily be associated with positive $\overline{v'q'}$. Such upgradient fluxes are associated with wave transience or dissipation.

While subrotating and superrotating jets appear equally likely at large L_D/a, recent calculations (Scott & Polvani, 2008) have shown that even at small L_D it is possible to ensure the emergence of superrotation, simply by including the physical mechanism of thermal relaxation. That the nature of the equatorial jets depends on details of energy dissipation is perhaps not surprising, in view of the above and the work of Andrews & McIntyre (1976) that demonstrated a sensitivity of mean flow changes at the equator to dissipation of Rossby-gravity waves. The example in Fig.3 illustrates a typical example of equatorial superrotation obtained in the presence of thermal dissipation, despite a small value of L_D/a.

5 Open questions: the nature of forcing and dissipation

The degree to which potential vorticity homogenisation occurs between jets will depend both on the way in which the mixing is forced and on the form of energy dissipation at large scales. What forms are used should be dictated by the physical situation of interest. On the gas giants, for example, the absence of a solid ground beneath the weather layers of these atmospheres together with measurements of radiative fluxes suggest that thermal relaxation is a more appropriate choice of dissipation than frictional damping.

The issue of forcing poses a major challenge. Potential vorticity mixing results from the motion of both coherent vortices and Rossby wave breaking. Random, isotropic forcing in spectral space, as was used in the experiments reported above, is convenient, but its effect on potential vorticity mixing may differ significantly from physical space forcing because of the loss of phase information.

Moreover, in a physical system, the zonal flows that arise from potential vorticity mixing will themselves affect the efficiency of a given forcing mechanism, for example by fixing the regions where waves may propagate and break. In a perfect potential vorticity staircase, waves are excluded from the regions between jets and may exist only on the discontinuities themselves, as edge waves (some instances can be discerned in Fig. 3). Forcing with a fixed rate of energy input, as is typically used in numerical studies of two-dimensional or geostrophic turbulence, and in the calculations presented here, ignores the natural adjustment of energy uptake by the flow as zonal jets intensify. In the terrestrial atmosphere the forcing of large-scale wave motions by topographic effects and land-sea temperature contrasts is relatively well-understood. On the gas giants, on the other hand, very little is known about the motion of the atmospheres at depth and the correct choice of forcing remains difficult to determine.

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